Hybrid Probabilistic Data Association and Variational Filtering for Multi-Target Tracking in Wireless Sensor Networks

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Abstract-A hybrid signal processing scheme is proposed for distributed multi-target tracking (MTT). For the sake of resource efficiency in a wireless sensor network (WSN), we reduce the problem to parallel cluster-based single target tracking when the targets are far apart, and switch to MTT only when data association becomes ambiguous. A sequential monte carlo method is employed to assign the ambiguous observations to specific targets or clutter, based on association probabilities. Whereas the rest observations are incorporated by the variational filter, which approximates the distribution of involved particles by a simple Gaussian distribution for each target. The natural and adaptive message compression dramatically reduces the resource consumption of the WSN. The low computation complexity also guarantees the one-line execution of the hybrid MTT scheme. In addition, experimental results prove that the proposed scheme succeeds in distinguishing and tracking multiple targets even during the occlusions.

I. INTRODUCTION

Among the potential applications of wireless sensor networks (WSNs), the tracking of mobile targets has found its major importance in monitoring wildlife animals, vehicles on the freeway, and surveillance in the battle field etc. [1]. Target tracking consists of recursively updating the posterior distribution of the target state given the sequence of sensor observations and the state evolution model [2]. Multi-target tracking (MTT) deals with state estimation of several moving targets, which is not a trivial extension of single target tracking but rather a challenging topic of research. The main difficulty of MTT comes from the assignment of a given measurement to a specific target.

Traditionally, the nearest neighbor (NN) approach, which utilizes the closest measurement to the predicted target measurement, is the simplest approach for MTT [3]. However, the NN measurements may be originated from a clutter, leading to filter divergence in many situations. As long as the data association is considered in a deterministic way, all possible associations must be exhaustively enumerated [4]. Multiple hypothesis tracking (MHT) [5] recursively builds all possible associations of measurements to existing/new tracks and false alarms, while respecting the mutual exclusion association constraint. MHT is capable of addressing the problems as low detection probability, high false alarm rates, delayed measurements, initiation and termination of tracks. However, it suffers from large storage space requirements, as the number of possible associations increases exponentially with time. The joint probabilistic data association filter (JPDAF) [6] is an alternative solution which consists of updating each individual track state with weighted combinations of all measurements. In fact, JPDAF is a particular way of combining the multiple hypotheses generated by MHT into a single hypothesis. Sequential Monte Carlo (SMC) method samples from complex association probability distribution conditioned on observations, where the sample with the highest probability is considered as the best association hypothesis [7], [8]. As the hypotheses are not explicitly enumerated, the large storage space is no longer required compared to MHT. Besides, the SMC method is very easy to implement and can be applied under very general hypotheses to cope with heavy clutters.

Due to the consideration of all possible events in the data association phase, MTT is an expensive task in terms of sensing, computation and communication. Concerning the extremely stringent resources in WSNs, an energy-aware distributed signal processing scheme is proposed in this paper. As the targets can travel arbitrarily and no a priori information on targets motion is provided, the general state evolution model proposed in [9], [10] is extended to describe the hidden states. Only the sensors which have detected the appearances of targets are activated to form data processing clusters for energy efficiency, where the cluster heads (CHs) are the ones with the most residual energy in each activated cluster. When the activated clusters are not overlapped, variational filters for single target tracking are parallelly executed in corresponding CHs. Otherwise, the activated CHs exchange ambiguous observations with each other, and invoke the probabilistic data association phase. A SMC method is employed to assign the ambiguous observations to specific targets or clutter based on the association probabilities. Whereas the variational tracking is delayed after the SMC phase to incorporate the rest of observations. Owing to the implicit compression of variational filtering, the temporal dependence of each target is reduce to a Gaussian distribution, which dramatically cuts off the inter-cluster communication. An overview of the hybrid MTT scheme is illustrated by Fig. 1.

The rest of the paper is organized as follows. In Section II, we formulate the variational tracking algorithm. Section III is dedicated to a detailed description of the SMC data association



Fig. 1: Block diagram of the hybrid MTT scheme

phase. In Section IV, performance of the proposed scheme is studied by simulations. Finally, we suggest future directions in Section V.

II. VARIATIONAL FILTERING

A. General state evolution model

The targets to track are modeled by independent Markovian dynamics. Let M be the number of targets, each component of the target temporal positions $X_t = \{x_t^j\}_{j=1}^M$ is assumed to evolve according to the following extended Markov model:

$$\begin{cases} \boldsymbol{x}_{t}^{j} \sim \mathcal{N}(\boldsymbol{x}_{t}^{j} | \boldsymbol{\mu}_{t}^{j}, \boldsymbol{\lambda}_{t}^{j}) \\ \boldsymbol{\mu}_{t}^{j} \sim \mathcal{N}(\boldsymbol{\mu}_{t}^{j} | \boldsymbol{\mu}_{t-1}^{j}, \bar{\boldsymbol{\lambda}}^{j}) \\ \boldsymbol{\lambda}_{t}^{j} \sim \mathcal{W}_{d}(\boldsymbol{\lambda}_{t}^{j} | \bar{\boldsymbol{V}}^{j}, \bar{n}^{j}) \end{cases}, \forall j = 1, \dots, M, \quad (1)$$

where $\bar{\lambda}^{j}$ is the initial precision matrix reflecting the uncertainty of the target position expectation at instant t with respect to the previous state. The state precision matrix λ_{t}^{j} is modeled by a d dimensional Wishart distribution, with \bar{V}^{j} and \bar{n}^{j} denoting respectively its precision matrix and degrees of freedom. Notice that $\bar{\tau}$ denotes initial fixed parameter. Assuming random mean and covariance for x_{t}^{j} leads to a probability distribution covering a wide range of tail behaviors, which allows discrete jumps in the target trajectory.

B. Observation model

The observation model depends on the sensing mode employed by the sensors. In this paper, it is assumed to be a range-based mode using the received signal strength indicator (RSSI) technology. The distance between a receiver and a transmitter is determined based on the knowledge of a pathloss model. Due to noisy wireless link, the received signal is corrupted by normally distributed error $\epsilon^i \sim \mathcal{N}(0, \sigma_i^{-2})$. The realistic measurements are formulated as follows:

$$y_t^{i,j} = \begin{cases} \| \boldsymbol{s}^i - \boldsymbol{x}_t^j \|, \text{if } RSSI \ge \gamma_s^i \\ 0, & \text{otherwise} \end{cases}$$

$$z_t^{i,j} = \beta^i y_t^{i,j} + \epsilon^i,$$

$$p(\boldsymbol{Z}_t | \boldsymbol{x}_t^j) = \prod_i \mathcal{N}(z_t^{i,j} | \beta^i y_t^{i,j}, \sigma_i^{-2}),$$
(2)

where β^i is the attenuation coefficient associated with the sensor *i*. *RSSI* defines the received signal power, which follows a path-loss function and is a one-to-one correspondence to the distance traveled by the signal. γ_s^i denotes the signal detection threshold of the sensor *i*, which is assumed to be identical for all the sensors.

C. Observation incorporation by variational Bayesian method

Variational filtering inherits many desirable properties from Bayesian Inference framework. Target tracking can be formulated as recursively estimating the predictive distribution as follows,

$$p(\mathbf{X}_t | \mathbf{Z}_{1:t-1}) = \int p(\mathbf{X}_t | \mathbf{X}_{t-1}) p(\mathbf{X}_{t-1} | \mathbf{Z}_{1:t-1}) d\mathbf{X}_{t-1}.$$
(3)

Based on the state evolution model $p(X_t|X_{t-1})$, the estimate of target states X_t is updated by incorporating the observation model $p(Z_t|X_t)$:

$$p(\mathbf{X}_t | \mathbf{Z}_{1:t}) = \frac{p(\mathbf{Z}_t | \mathbf{X}_t) p(\mathbf{X}_t | \mathbf{Z}_{1:t-1})}{p(\mathbf{Z}_t | \mathbf{Z}_{1:t-1})}.$$
 (4)

Without loss of generality, the j^{th} target state \boldsymbol{x}_t^j is extended to an augmented state $\boldsymbol{\alpha}_t^j = (\boldsymbol{x}_t^j, \boldsymbol{\mu}_t^j, \boldsymbol{\lambda}_t^j)$ by Eq. (1), the distribution of interest takes the form of a marginal posterior distribution $p(\boldsymbol{\alpha}_t^j | \boldsymbol{Z}_{1:t})$. A variational Bayesian method is proposed for approximating the intractable integrals arising in Bayesian inference. Introducing a separable distribution $q(\boldsymbol{\alpha}_t^j)$, an analytical approximation to the parameter posterior probability $p(\boldsymbol{\alpha}_t^j | \boldsymbol{Z}_{1:t})$ is provided by minimizing the Kullback-Leibler divergence:

$$D_{\mathrm{KL}}(q||p) = \int q(\boldsymbol{\alpha}_t^j) \log \frac{q(\boldsymbol{\alpha}_t^j)}{p(\boldsymbol{\alpha}_t^j|\boldsymbol{Z}_{1:t})} d\boldsymbol{\alpha}_t^j$$

where $q(\boldsymbol{\alpha}_t^j) = q(\boldsymbol{x}_t^j)q(\boldsymbol{\mu}_t^j)q(\boldsymbol{\lambda}_t^j).$

To minimize D_{KL} subject to the constraint $\int q(\alpha_t) d\alpha_t = 1$, Lagrange multiplier is used, yielding the following approximate distribution [9],

$$q(\boldsymbol{x}_t^j) \propto \exp \langle \log p(\boldsymbol{Z}_{1:t}, \boldsymbol{\alpha}_t) \rangle_{\prod q(\boldsymbol{\mu}_t^j)q(\boldsymbol{\lambda}_t^j)}$$
(5)

where $\langle \cdot \rangle_q$ denotes the expectation operator relative to the distribution q. Taking into account the separable approximate distribution $q(\alpha_{t-1}^j)$, the predictive distribution $p(\alpha_t^j | \mathbf{Z}_{1:t-1})$ and the filtering distribution $p(\alpha_t^j | \mathbf{Z}_{1:t})$ are sequentially approximated according to the following scheme:

$$p(\boldsymbol{\alpha}_t^j | \boldsymbol{Z}_{1:t-1}) \propto p(\boldsymbol{x}_t^j, \boldsymbol{\lambda}_t^j | \boldsymbol{\mu}_t^j) q_p(\boldsymbol{\mu}_t^j)$$
(6)

$$p(\boldsymbol{\alpha}_{t}^{j}|\boldsymbol{Z}_{1:t}) \propto p(\boldsymbol{Z}_{t}|\boldsymbol{x}_{t}^{j})p(\boldsymbol{x}_{t}^{j},\boldsymbol{\lambda}_{t}^{j}|\boldsymbol{\mu}_{t}^{j})q_{p}(\boldsymbol{\mu}_{t}^{j}), \quad (7)$$

here $q_{p}(\boldsymbol{\mu}_{t}^{j}) = \int p(\boldsymbol{\mu}_{t}^{j}|\boldsymbol{\mu}_{t-1}^{j})q(\boldsymbol{\mu}_{t-1}^{j})d\boldsymbol{\mu}_{t-1}^{j}.$

Therefore, through a simple integral with respect to $\boldsymbol{\mu}_{t-1}^{j}$, the distributions involved in the Bayesian inference can be sequentially updated. The temporal dependence is hence reduced to the incorporation of only one Gaussian component approximation $q(\boldsymbol{\mu}_{t-1}^{j})$ for the target j. As variational calculus leads to closed-form expressions of $q(\boldsymbol{\mu}_{t}^{j})$ and $q(\boldsymbol{\lambda}_{t}^{j})$ [9], [10], the expectations involved in the predictive distribution $p(\boldsymbol{\alpha}_{t}^{j}|\boldsymbol{Z}_{1:t-1})$ thus have closed forms. However, due to observation incorporation, the estimate of target state \boldsymbol{x}_{t}^{j} does not have a tractable form. By combining the Eq. (5) and (6), we have the following form,

$$q(\boldsymbol{x}_t^j) \propto p(\boldsymbol{Z}_t | \boldsymbol{x}_t^j) \mathcal{N}(\langle \boldsymbol{\mu}_t^j \rangle, \langle \boldsymbol{\lambda}_t^j \rangle).$$
(8)

Thus the state evolution model (1) and the observation model (2) are incorporated to update $q(x_t^j)$. This form immediately suggests an Importance Sampling procedure:

$$\boldsymbol{x}_{t}^{j,(i)} \sim \mathcal{N}(\langle \boldsymbol{\mu}_{t}^{j} \rangle, \langle \boldsymbol{\lambda}_{t}^{j} \rangle), \ \boldsymbol{x}_{t}^{(i)} = \{\boldsymbol{x}_{t}^{j,(i)}\}_{j=1}^{M},$$
$$\boldsymbol{w}_{t}^{(i)} \propto p(\boldsymbol{Z}_{t} | \boldsymbol{x}_{t}^{(i)}), \ \langle \boldsymbol{X}_{t} \rangle = \sum_{i=1}^{N} \boldsymbol{w}_{t}^{(i)} \boldsymbol{x}_{t}^{(i)}, \tag{9}$$

III. PROBABILISTIC DATA ASSOCIATION

We assume that only the sensors within the overlapped area of clusters can detect more than one targets at the same time. Their observations are also much more vulnerable to collisions and clutters in the wireless links. We use $Z_t^{amb} =$ $\{z_t^{i_{amb}}\}_{i_{amb}=1}^{N_{amb}}$ to denote the set of measurements observed by these sensors, which is composed of detection measurements and clutter measurements, where the latter are assumed to be uniformly distributed in the observation area. As we do not know the origin of each measurement $z_t^{i_{amb}}$, one has to introduce a vector K_t to describe the associations between the measurements and the targets. Each component of K_t is a random variable that takes value among $\{0, \ldots, M\}$, where 0 denotes the clutter. Assuming the total amount of ambiguous observations is N_{amb} , accordingly, $K_t^{amb} = \{K_t^{i_{amb}}\}_{i_{amb}=1}^{N_{amb}}$ where $K_t^{i_{amb}} = j$ indicates that $z_t^{i_{amb}}$ is associated with the target j. In this case, $z_t^{i_{amb}}$ is a realization of the stochastic process:

$$z_t^{i_{amb}} = H_t^{j,i_{amb}}(x_t^j, \sigma_t^{i_{amb}}).$$
 (10)

The noise $\sigma_t^{i_{amb}}$ is supposed to be a white noise independent of the observation noises. We assume that the hypothesis $H_t^{j,i_{amb}}$ can be associated with a functional form $F(z_t^{i_{amb}}; \boldsymbol{x}_t^j)$ such that

$$F(z_t^{i_{amb}}; \boldsymbol{x}_t^j) \propto p(z_t^{i_{amb}} | \boldsymbol{x}_t^j, K_t^{i_{amb}} = j).$$
(11)

If $K_t^{i_{amb}} = 0$, the measurement $y_t^{i_{amb}}$ is associated with the clutter. As the indexing of the ambiguous measurements is arbitrary, all the measurements have the same *a priori* probability to be associated with a given target *j*. For each ambiguous measurement, a vector $\pi_t = {\{\pi_t^j\}_{j=0}^M \in [0,1]^{M+1}}$ is defined for the association probability, where π_t^j is a discrete probability that any measurement is associated with the target *j*. To solve the data association problem, the following assumptions are commonly made [11]:

- 1) One measurement can originate from one target or from the clutter.
- One target can produce zero or one measurement at one time.

The first assumption expresses that the association is exclusive and exhaustive, namely $\sum_{j=0}^{M} \pi_t^j = 1$. The second assumption implies that N_{amb} may differ from M and, above all, that the association variables $K_t^{i_{amb}}$ for $i_{amb} = 1, \ldots, N_{amb}$ are dependent.

The number of clutter measurements is assumed to arise from a Poisson density of parameter aS, where S is the size of the observation area, and a is the number of false alarms per area unit. The association probability π_t^0 that a measurement is associated with the clutter, is a constant that can be computed as follows,

$$\pi_t^0 = \sum_{l=0}^{N_{amb}} P(K_t^{i_{amb}} = 0 | N_t^0 = l) P(N_t^0 = l)$$
$$= \sum_{l=0}^{N_{amb}} \frac{l}{N_{amb}} \exp(-aS) \frac{(aS)^l}{l!}, \qquad (12)$$

where N_t^0 is the number of measurements arising from the clutter at time t. Assuming that there are l clutter originated measurements among the N_{amb} measurements, the a priori probability that any measurement comes from the clutter is equal to l/N_{amb} . Thus, we get the equality $P(K_t^{i_{amb}} = 0|N_t^0 = l) = l/N_{amb}$.

At instant t, if $N_{amb} > 0$, the data association phase is invoked, which is initialized by generating a set of N particles $X_t = \{x_t^{(i)}, w_t^{(i)}\}_{i=1}^N$. For all i = 1, ..., N, the likelihood of the particles are formulated as:

$$p(\boldsymbol{Z}_{t}^{amb}|\boldsymbol{x}_{t}^{(i)}) = \prod_{i_{amb}=1}^{N_{amb}} p(z_{t}^{i_{amb}}|\boldsymbol{x}_{t}^{(i)})$$
(13)
$$\propto \prod_{i_{amb}=1}^{N_{amb}} [\frac{\pi_{t}^{0}}{S} + \sum_{j=1}^{M} F(z_{t}^{i_{amb}}; \boldsymbol{x}_{t}^{j,(i)}) \pi_{t}^{j}].$$

The vectors X_t , K_t and π_t are considered to be random variables with known prior densities. Samples are then obtained iteratively from their joint posterior using a proper Markov Chain Monte Carlo (MCMC) technique, namely, the Gibbs sampler [7]. Let $\Theta_t = (X_t, K_t, \pi_t)$, the Gibbs algorithm consists of generating a Markov chain that converges to the distribution $p(\Theta_t | Z_t^{amb})$, which cannot be sampled directly. In order to implement the Gibbs sampler, we choose the following partition:

$$\begin{cases} \Theta_{t}^{i} = K_{t}^{i}, \text{ for } i = 1, \dots, N_{amb} \\ \Theta_{t}^{N_{amb}+j} = \pi_{t}^{j}, \text{ for } j = 1, \dots, M \\ \Theta_{t}^{N_{amb}+M+j} = \mathbf{x}_{t}^{j}, \text{ for } j = 1, \dots, M \end{cases}$$
(14)

The initialization of the Gibbs sampler consists of assigning uniform association probabilities, i.e., $\pi_t^j = (1 - \pi_t^0)/M$, and taking the predictive target states made by Eq.(??) to initialize $X_t = \langle X_t \rangle_{q_{t|t-1}}$. The K_t variables do not need initialization, which are sampled conditioned on π_t and X_t at the first step of the Gibbs sampler. After a finite number of iterations, estimations of the random variables are obtained, namely $\Theta_t = (X_t, K_t, \hat{\pi}_t)$. Therefore, the ambiguous observations are assigned by K_t and are incorporated to update the target estimates \hat{X}_t . The particles $\{x_t^{(i)}, w_t^{(i)}\}_{i=1}^N$ generated by the data association phase are directly employed to incorporate the rest of observations $Z_t \setminus Z_t^{N_{amb}}$, according to (9). After the run of the variational filtering algorithm, estimates on the targets are refined, especially the distributions of the particles are naturally approximated by a simple Gaussian distribution $q(\boldsymbol{\mu}_t^j)$ for each of the target.

IV. SIMULATIONS

In the simulations, to ensure the 4-coverage condition [12], 400 sensors were assumed to be uniformly deployed in a 2 dimensional field $(100 \times 100 \ m^2)$, and their sensing ranges were identically fixed to 10 m. Two targets of different velocities and distinct trajectories were tracked, where both the trajectories were of the same duration of 100 s. Performance of the proposed scheme is shown in Fig. 2, where accurate tracking performances are achieved. It is shown in Fig. 3 that the hybrid MTT scheme succeeded in tracking the targets separately. When the two targets encountered each other (between t = 62 and t = 88), the tracking performances degrade because of data association ambiguity. However, the performance degradations (maximal estimate error for the target A is 1.3043 and that for the target B is 1.7612) are still acceptable. Owing to the diversity of the general state evolution model, the target states are successfully described despite of their distinctions, which leads to similar tracking performances of both targets in simulations. Monte Carlo simulations were performed on the same configuration, whose results are reported in Table I. The tracking accuracy is evaluated by the average Root Mean Square Error (RMSE) of the 100 runs of Monte Carlo simulations. Notice that the execution time is evaluated by the average time consumed per sampling slot. Compared to the sampling slot 1s, the average execution time of 0.1181sguarantees the on-line implementation of our scheme.



Fig. 2: Multi-target tracking performance

Evaluation	Target A	Target B
Tracking accuracy	0.1621	0.1850
Maximal estimate error (m)	1.3043	1.7612
Execution time for MTT (s)	0.1181	

TABLE I: Evaluation of the hybrid MTT Scheme



Fig. 3: Root Mean Square Error of MTT

V. PERSPECTIVES

However, in this study, the number of targets to be tracked is assumed to be known *a priori*, which may not be reasonable in real situations. Therefore, we are thinking of integrating the hypothesis test into the hybrid scheme, to deal with the cases of arrivals of new targets and disappearances of the tracked targets.

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