VARIATIONAL FILTERING ALGORITHM FOR INTERDEPENDENT TARGET TRACKING AND SENSOR LOCALIZATION IN WIRELESS SENSOR NETWORK

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ABSTRACT

A novel algorithm for interdependent sensor localization and target tracking in wireless sensor networks is proposed in this paper. Based on range measurements between sensors and the target, sensor location estimations and that of the target are interdependently improved. The contribution of this work lies in three aspects: first, the algorithm is executed on a fully decentralized cluster scheme to reduce the energy and bandwidth consumption; second, a general state evolution model is proposed to describe the target and activated sensors, since no a priori information of the the target motion is available; finally, the variational method further lightens the communication burden and terminates the error propagation problem. The effectiveness of the proposed algorithm is evaluated and compared in terms of tracking accuracy, localization precision and execution time.

1. INTRODUCTION

Many strategic applications of Wireless Sensor Networks (WSN), especially the target detection, surveillance and tracking, require that the sensor locations are known [1, 2, 3]. However, it is impractical to equip each sensor with a GPS unit in view of the configuration cost. On the other hand, the sensor hardware is generally cheaper than the cost of accurate sensor placement [4]. Random deployment is thus more desirable especially in military applications. As a result, sensor localization research has gained a lot of attention. In earlier works, sensor localization and target tracking are always treated as two separate phases. But in fact, the two phases can be complementary to one another. By incorporating measurement information between sensors and the target, estimations of sensors and the target can be interdependently improved. This is an attractive solution for several reasons: first, it poses no additional requirements on both the deployment phase and sensor hardware; second, it allows the precision of sensor localization to be continuously improved, even during the tracking phase.

In this paper we propose a novel Variational Filtering algorithm for interdependent target tracking and sensor localization. The rest of the paper is organized as follows: in section 2 we briefly summarize related existing works. The detection model and the general state evolution model are formulated in section 3. In section 4, the variational filtering algorithm for interdependent sensor localization and target tracking (VFISLTT) is presented in detail. Finally, the proposed schemes are evaluated by simulations in section 5. Section 6 concludes the paper.

2. RELATED WORK

There have been some previous works using a mobile object to assist in localizing sensors [5, 6, 7]. Most of them require that the position of the mobile be known. Consequently, the mobile must be constrained to a well-known trajectory or be equipped with a GPS-like system. One exception is [6], which builds a constraint structure as measurements become available. Compared to [6], we employ an extensible statistical model to incorporate measurements.

Simultaneous localization and mapping (SLAM) is the process by which a mobile robot can build a map of the environment and, at the same time, use this map to compute its own location [8]. The past decade has seen rapid and exciting progress in solving the SLAM problem and many compelling implementations. But it requires the control signal for the mobile robot to be known, which simplifies the problem. Another problem labeled as Simultaneous Localization And Tracking (SLAT) in [2], is analogous to our algorithm. By incorporating the range measurements in a Bayesian framework, a joint probability distribution over the active sensors' positions and the trajectory of the mobile are updated together. As mentioned above, to avoid representation complexity and unnecessary consumption of energy and bandwidth in the WSN, we use the Variational method to approximate the state with an extended Gaussian distribution. Furthermore, error propagation is terminated in the VFISLTT, which is always unavoidable in other approximation methods, such as KD-tree [9] or GMM [10].

3. PROBLEM FORMULATION

3.1 Detection Model

Sensor network localization algorithms mainly rely on range measurement, angle measurement, neighborhood proximity or hop count methods. In this paper, we employ a realistic range measurement detection model, where range measurements are corrupted by Gaussian distributed errors:

$$y_{i,x}^t = \begin{cases} \parallel s_i - x^t \parallel + \varepsilon_i, & \text{if } y_{i,x}^t \leq \gamma_i \\ 0, & \text{otherwise} \end{cases},$$

$$\begin{aligned} \text{with} \quad & \varepsilon_i \sim N(0, \sigma_i^2) \\ \text{and} \quad & y_{i,s} = \{y_{i,j} | y_{i,j} = \parallel s_i - s_j \parallel + \varepsilon_i\}, \\ \text{where} \quad & j \in Neighbor_i, \\ & y_i^t = \{y_{i,x}^t, y_{i,s}\} \end{aligned} \tag{1}$$

where $y_{i,x}^t$ denotes the detected Euclidean distance between the sensor i and the target x at instant t, γ_i is the sensor detection radius, and ε_i denotes the detection error of the sensor

i. Similarly, $\boldsymbol{y}_{i,s}$ denotes the set of Euclidean distance measurements between the sensor *i* and its neighbor sensors They are corrupted by the detection error ε_i too. $\boldsymbol{y}_i^t$ gathers all observations of the sensor *i* at instant *t*. To minimize energy and bandwidth consumption, only those sensors, which have detected the presence of the target, exchange information between each other. Thus an activated cluster is formed to perform the Variational Filtering algorithm for Interdependent Sensor Localization and Target Tracking (VFISLTT).

3.2 General State Evolution Model

Since the mobile target travels arbitrarily in the sensor field. Instead of a traditional kinematic parameter model [11, 12, 13], we employ the general state evolution model [14, 15, 16], which is more adaptive to practical situation and has no restriction on velocity and moving direction of the target. Considering a 2D space, at instant *t*, the hidden state to be estimated contains the target position $\boldsymbol{x}^t$ and a set of activated sensor positions $\boldsymbol{S}^t = \{\boldsymbol{s}_1^t, \boldsymbol{s}_2^t, \ldots, \boldsymbol{s}_m^t\}$, where *m* denotes the number of sensors that have detected the target. Take a sensor *i* for example, $\boldsymbol{s}_i$ is assumed to be a Gaussian variable, whose expectation is its initial deployment value $\overline{\boldsymbol{s}}_i$, and the precision matrix η_i indicates its position offset due to deployment error and other spatial factors. The target $\boldsymbol{x}^t$ is assumed to be Gaussian distributed, with a random mean μ^t and a random precision matrix λ^t to further capture the uncertainty of the state distribution:

$$\begin{cases} \boldsymbol{s}_i & \sim & N(\overline{\boldsymbol{s}}_i, \eta_i) \\ \boldsymbol{x}^t & \sim & N(\mu^t, \lambda^t) \\ \mu^t & \sim & N(\mu^{t-1}, \overline{\lambda}) \\ \lambda^t & \sim & W_d(\overline{\boldsymbol{V}}, \overline{n}) \end{cases} \qquad (2)$$

where $\overline{\lambda}$ is the initial precision matrix to reflect the uncertainty for estimation of the target position at instant *t* versus that of the former instant. The state precision matrix is modeled by a *d* dimensional Wishart distribution, with $\overline{\boldsymbol{V}}$ and $\overline{n}$ denoting respectively the precision matrix and the degrees of freedom. Note that $\overline{\cdot}$ denotes the initial fixed parameter.

4. VFISLTT ALGORITHM

Given the general state evolution model described above, the hidden state has been extended to $(\boldsymbol{x}^t, \mu^t, \lambda^t, \boldsymbol{S}^t)$. We use α^t to denote them, thus the distribution of interest takes the form of posterior distribution $p(\alpha^t|\boldsymbol{y}^{1:t})$, where $\boldsymbol{y}^{1:t}$ denotes range measurements gathered previously. The variational approach consists in approximating $p(\alpha^t|\boldsymbol{y}^{1:t})$ by a separable distribution $q(\alpha^t)$, which minimizes the Kullback-Leibler (KL) divergence error:

$$D_{KL}(q||p) = \int q(\alpha^t) \log \frac{q(\alpha^t)}{p(\alpha^t|\boldsymbol{y}^{1:t})} d\alpha^t,$$
where $q(\alpha^t) = \prod_i q(\alpha_i) = q(\boldsymbol{x}^t)q(\mu^t)q(\lambda^t)q(\boldsymbol{S}^t)$
and $q(\boldsymbol{S}^t) = \prod_{i=1}^m q(\boldsymbol{s}_i^t)$.

With variational calculus, the following approximate distribution yields,

$$q(\alpha_i) \propto \exp\langle \log p(\boldsymbol{y}^{1:t}, \alpha^t)\rangle_{\prod_{j \neq i} q(\alpha_j)} \qquad (3)$$

where $\{\alpha_i\}$ denotes the subsets of α^t, which are $\boldsymbol{x}^t, \mu^t, \lambda^t$ and $\boldsymbol{S}^t$. Note that $\langle . \rangle_q$ denotes the expectation operator relative to the distribution *q*.

Taking into account the separable approximate distribution at time $t-1$, the filtering distribution at time *t* is written,

$$p(\alpha^t|\boldsymbol{y}^{1:t}) \propto p(\boldsymbol{y}^t|\boldsymbol{x}^t, \boldsymbol{S}^t)p(\boldsymbol{x}^t|\mu^t, \lambda^t)p(\lambda^t)q_p(\mu^t), \qquad (4)$$

$$\text{with} \quad q_p(\mu^t) = \int p(\mu^t|\mu^{t-1})q(\mu^{t-1})d\mu^{t-1}.$$

The temporal dependence on the past is hence reduced to incorporate only one component approximation $q(\mu^{t-1})$. The communication between two successive active clusters is then reduced to sending the mean and the precision matrix of $q(\mu^{t-1})$. Since it turns out to be a Gaussian distribution. The approximate distribution yields thus a natural and adaptive compression of the filtering distribution, which is propagated in the sensor network without lossy compression.

With a view to the general state evolution model in formulation (2), the inference leads to the mean and the precision matrix of the target state in the following form:

$$q(\mu^t) \sim N(\mu_*^t, \lambda_*^t), \quad q(\lambda^t) \sim W_d(\boldsymbol{V}_*^t, n_*)$$

By incorporating the equation (3), the parameters above are iteratively updated according to the following scheme:

$$\begin{aligned} \mu_*^t &= (\lambda_*^t)^{-1}(\langle\lambda^t\rangle\langle\boldsymbol{x}^t\rangle + \lambda_p^t\mu_p^t) \\ \lambda_*^t &= \langle\lambda^t\rangle + \lambda_p^t \\ n_* &= \overline{n}+1 \\ \boldsymbol{V}_*^t &= (\langle\boldsymbol{x}^t\boldsymbol{x}^{tT}\rangle - \langle\boldsymbol{x}^t\rangle\langle\mu^t\rangle^T - \langle\mu^t\rangle\langle\boldsymbol{x}^t\rangle^T + \langle\mu^t\mu^{tT}\rangle + \overline{V}^{-1})^{-1} \\ \mu_p^t &= \mu_*^{t-1} \\ \lambda_p^t &= ((\lambda_*^{t-1})^{-1} + \overline{\lambda}^{-1})^{-1} \end{aligned}$$
$$(5)$$
$$(6)$$

However, the target state $\boldsymbol{x}^t$ and sensors' positions $\boldsymbol{S}^t$ do not have closed forms of approximate distribution. In order to compute their means and covariances, we employ the Importance Sampling Scheme, where samples are drawn from an Gaussian distribution and are weighted according to their likelihoods. By combining the equation (3) and (4), the likelihood expression for $q(\boldsymbol{x}^t)$ and $q(\boldsymbol{s}_i^t)$ have the following forms:

$$q(\boldsymbol{x}^t) \propto \prod_{i=1}^m p(y_{i,x}^t|\boldsymbol{x}^t, \boldsymbol{s}_i^t)N(\boldsymbol{x}^t|\langle\mu^t\rangle, \langle\lambda^t\rangle)$$
$$q(\boldsymbol{s}_i^t) \propto p(y_{i,x}^t|\boldsymbol{x}^t, \boldsymbol{s}_i^t)\prod_{j \neq i}^{m-1} p(y_{i,j}^t|\boldsymbol{s}_i^t, \boldsymbol{s}_j^t)N(\boldsymbol{s}_i^t|\overline{\boldsymbol{s}}_i, \eta_i)$$

Therefore, the estimations of the target state and the sensor positions are interdependently updated by incorporating the detection model and the general state evolution model. Generally speaking, the sensor locations need to be known as a priori for precise target tracking. Thus, a sub-program is launched to initially localize the activated sensors by the classical particle filtering algorithm. Taking sensor *i* for example:

$$\boldsymbol{s}_{i,(k)}^t \sim N(\overline{\boldsymbol{s}}_i, \eta_i), \ w_{i,(k)}^t \propto \prod_{j \neq i}^{m-1} p(y_{i,j}^t|\boldsymbol{s}_i^t, \boldsymbol{s}_j^t) \qquad (7)$$

After the initial sensor localization phase, the target state and the activated sensor location estimations are interdependently updated:

$$\boldsymbol{x}_{(k)}^t \sim N(\langle\mu^t\rangle, \langle\lambda^t\rangle), \ w_{x,(k)}^t \propto \prod_{i=1}^m p(y_{i,x}^t|\boldsymbol{x}^t, \boldsymbol{s}_i^t)$$
$$\boldsymbol{s}_{i,(k)}^t \sim N(\overline{\boldsymbol{s}}_i, \eta_i), \ w_{i,(k)}^t \propto p(y_{i,x}^t|\boldsymbol{x}^t, \boldsymbol{s}_i^t)\prod_{j \neq i}^{m-1} p(y_{i,j}^t|\boldsymbol{s}_i^t, \boldsymbol{s}_j^t)$$

Therefore, by incorporating the variational method and the importance sampling scheme, estimations for the activated sensors and the target are interdependently improved.

Pseudocode of the VFISLTT algorithm is summarized in Algorithm 1.

Algorithm 1: VFISLTT algorithm

Input: $\overline{S}$, $\overline{\lambda}$, $\overline{V}$, $\overline{n}$, μ_*^0, λ_*^0
Output: $\langle x_t \rangle$, $\langle S^t \rangle$

1   **for** $t = 1, 2, \ldots,$ **do**
2   **for** $i=1 : m$ **do**
3   Generate N samples $\{s_{i,(k)}^t, w_{i,(k)}^t\}_{k=1}^N$ from $N(\overline{s}_i, \eta_i)$;
4   Compute the initial expectation $\langle s_i^t \rangle$ and corresponding precision matrix according to the equation(7) ;
5   **end**
6     $\mu_p^t = \mu_*^{t-1}$, $\lambda_p^t = ((\lambda_*^{t-1})^{-1} + \overline{\lambda}^{-1})^{-1}$, $q_p(\mu^t) = N(\mu_p^t, \lambda_p^t) = \int p(\mu^t|\mu^{t-1})q(d\mu^{t-1})$;
7   Initiate $\mu_*^t = \mu_p^t$, $\lambda_*^t = 2\lambda_p^t$, $n_*^t = \overline{n}+1$, $V_*^t = (2\lambda_{p-1}^t + \overline{V}^{-1})^{-1}$;
8   Calculate the initial expectations $\langle \mu^t \rangle = \mu_*^t$ and $\langle \lambda^t \rangle = n_*^t V_*^t$;
9   **while** *not converge* **do**
10   Generate N samples $\{x_{(k)}^t, w_{(k)}^t\}_{k=1}^N$ from $q(x^t)$, where $q(x^t) \propto \prod_{i=1}^m p(y_{i,x}^t|x^t, s_i^t)N(\langle \mu^t \rangle, \langle \lambda^t \rangle)$;
11   Compute $\langle x^t \rangle = \sum_{k=1}^N w_{(k)}^t x_{(k)}^t$ and corresponding precision matrix ;
12   **for** $i=1 : m$ **do**
13   Generate N samples $\{s_{i,(k)}^t, w_{i,(k)}^t\}_{k=1}^N$ from $q(s_i^t)$, where $q(s_i^t) \propto$ $p(y_{i,x}^t|x^t, s_i^t)\prod_{j\neq i}^{m-1} p(y_{i,j}^t|s_i^t, s_j^t)N(s_i^t|\overline{s}_i, \eta_i)$;
14   Update the expectation $\langle s_i^t \rangle$ and corresponding precision matrix;
15   **end**
16   Update the variational parameters μ_*^t, λ_*^t, n_*^t, V_t^* according to the equation (5);
17   Re-update the expectations $\langle \mu^t \rangle$ and $\langle \lambda^t \rangle$;
18   **end**
19   **if** *hand-off operation between* CHs *happens* **then**
20   Communicate $q(\mu^t) \sim N(\mu_*^t, \lambda_*^t)$;
21   **else**
22   Replace the storage of particles in the CH by μ_*^t and λ_*^t;
23   **end**
24   Return the target position estimation $\langle x^t \rangle$ and those of the activated sensors: $\langle S^t \rangle = \{\langle s_1^t \rangle, \langle s_2^t \rangle, \ldots, \langle s_m^t \rangle\}$;
25 **end**

Fig. 1 demonstrates the Flowchart of the VFISLTT algorithm. Apparently the VFISLTT algorithm is divided to three sub-programs:

- Cluster activation and formation
- Initial cluster sensors localization
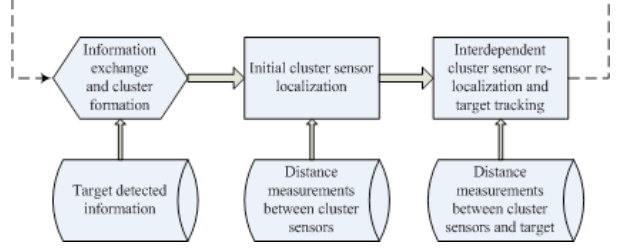- Interdependent cluster sensors re-localization and target tracking

Figure 1: Flowchart of the VFISLTT Algorithm

Please note that only the activated sensors are localized at each instant. Therefore, sensors close to high traffic areas can be continuously localized, and in turn, facilitates target tracking with high precision.

5. EVALUATION AND SIMULATION

The performance of the VFISLTT algorithm is quantified by three criteria:

- tracking accuracy: the mean square error (MSE) between the estimation and the true trajectory of the mobile target
- localization precision: the MSE between the estimations and the sensor positions
- execution time

Considering a target tracking duration of 320 time slots, 400 sensors are initially set to be uniformly deployed in a 2 dimensional field ($100 \times 100 m^2$). The sensor detection radius is set to 9 m to ensure coverage condition. Due to the spatially varying environment factors and deployment errors, sensors are in fact randomly distributed around their initially set locations $\overline{S} = \{\overline{s}_1, \overline{s}_2, \ldots, \overline{s}_{400}\}$. We assumed that $s_i \sim N(\overline{s}_i, \eta_i)$, where η_i are identical for all the sensors. These parameters involved in the general state evolution model (2) are initially set as:

$$\overline{V} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, \qquad \eta_i = \begin{bmatrix} 1/5 & 0 \\ 0 & 1/5 \end{bmatrix},$$

$$\overline{\lambda} = \begin{bmatrix} 1/1000 & 0 \\ 0 & 1/1000 \end{bmatrix}, \quad \overline{n} = 1$$

The low state precision $\overline{\lambda}$ and the degree of freedom $\overline{n}$ allow a general non informative prior. The tracking performance of the VFISLTT algorithm is evaluated by the Mean Square Error (MSE) in Fig. 2. We can find the error distributions of the sensor deployment and the localization of them in Fig. 3. It shows that sensor deployment errors are Gaussian distributed, after the execution of the VFISLTT algorithm, most of the sensors are localized within a much smaller error. Fig. 4 compares the MSE of sensor deployment and their localization. Note that only sensors, which have detected presence of the target, are localized by the VFISLTT algorithm.

We also compare our VFISLTT algorithm with the traditional strategy, which localizes all the sensors in the field and then tracks the target based on the sensor location estimations. In the traditional strategy, we deploy the classical particle filtering algorithm to localize the sensors, then variational method and importance sampling scheme are used to track the target. By repeatedly executing the two algorithms on the same configuration, the average values of the three criteria are obtained to evaluate their performance. Table.1
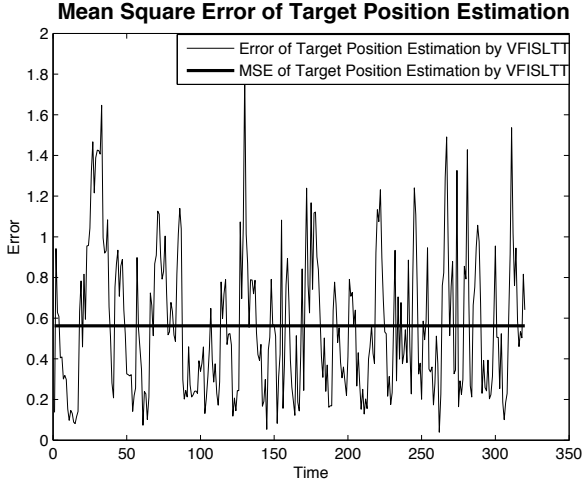
Mean Square Error of Target Position Estimation

Figure 2: Tracking Performance of the VFISLTT Algorithm
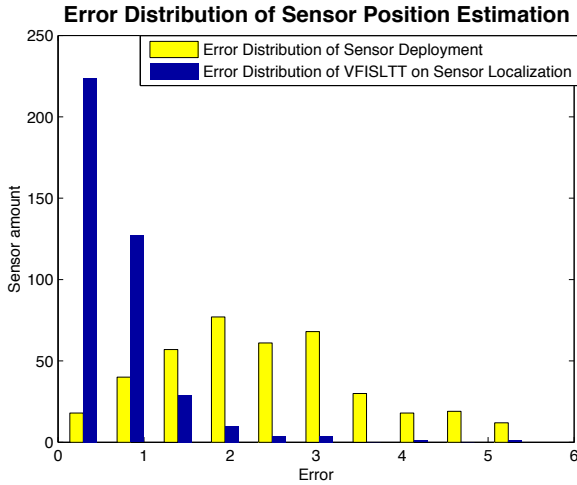
Error Distribution of Sensor Position Estimation

Figure 3: Error Distributions Comparison between the Sensor Deployment & the Sensor Localization

shows that the VFISLTT algorithm outperforms the traditional one on interdependently and continuously improving estimates of both the sensors and the target. The tracking accuracy and the localization precision are quantized in MSE.

Evaluation	Traditional	VFISLTT
Tracking accuracy	11.5661	1.4176
Localization precision	8.59	1.19
Execution time (s)	0.0700	0.0624

Table 1: Evaluation of Traditional and VFISLTT Algorithms

6. CONCLUSION

A variational filtering algorithm for interdependent sensor localization and target tracking has been proposed in the context of WSN. Since no a priori information on the target and the activated sensors is available, VFISLTT algorithm aimed at continuously updating and improving the estimation of

Figure 4: MSE Comparison between the Sensor Deployment & the Sensor Localization

sensor positions and that of the target. To minimize resource consumption in WSN, the algorithm is executed on a fully decentralized cluster scheme. That is to say, only sensors which have detected the target are activated to form a cluster in order to process data. The variational method allows an implicit compression of the exchanged statistics between clusters. It further reduces energy and bandwidth consumption, while terminate the essential error propagation problem, which is always unavoidable in other approximation methods. As the target can travel arbitrarily in WSN, and location information of the activated sensors is not accurate too, a general state evolution model is proposed to describe the hidden state, which is more adaptable to the non-linear/ non-gaussian situation than the other kinematic parameter model. In conclusion, as the target move freely in WSN, a large number of range measurements are generated, which facilitate both the localization of the activated sensors and the target tracking. By incorporating these measurements into the VFISLTT algorithm, estimations of sensors and that of the target are interdependently and continuously improved.

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