# Interval-based localization for mobile sensors in low-anchors density networks

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Abstract—In this article, we propose an original approach for self-localization in mobile sensor networks. The proposed approach is developed for low-anchors density networks. Based on intervals theory, the presented method is an online technique yielding a bounded-cumulative error. The estimation of the positions of mobile sensors is performed using multi-hop observation model added to an *a priori* mobility model. One of the contributions of this paper is that it uses the measurements of all types of sensors, including those that do not have GPS, denoted *non-anchor nodes*. Compared to the existing localization techniques, this method leads to a higher accuracy with a low computational cost.

*Index Terms*—Distributed estimation, interval analysis, mobile sensors, nonlinear estimation, signal processing

#### I. INTRODUCTION

Recently, Wireless Sensor Networks (WSNs) have become a growing and challenging research field of signal processing domain. They are composed of a large number of tiny lowcost wireless sensors [1]. These smart devices are limited in computational capacities and memory resources. The major constraint of these sensors is their low energy consumption since they are equipped with non-renewable batteries. In order to extend the lifetime of the networks, all built-in algorithms must focus on energy conservation.

Sensor networks have been applied in different fields of applications such as military and biomedical analysis [2], [3]. Due to the lack of a fixed infrastructure in these networks, the sensors are able to move in an uncontrolled manner, leading to Mobile Sensor Networks (MSNs). Since sensed data are related to the location of the sensors in almost all applications of MSNs, the localization problem has become a fundamental issue. A direct way to obtain the nodes locations is to equip all sensors with Global Positioning Systems (GPS) [4]. However, this solution is not practical since GPS receivers are high energy consuming. An alternative solution consists of considering two types of sensors: anchors and non-anchor nodes. Anchors are equipped with GPS receivers, and thus know their locations. Non-anchor nodes, or simply nodes, have unknown positions, and thus they are localized using location information of anchors. Many anchor-based algorithms have been proposed in literature, such as Monte Carlo-based approaches [5]. These techniques consist of generating particles in order to cover the solution area. The main disadvantage of these methods is their high consumption of memory resources.

Other methods based on intervals have been proposed. For instance, in [6], Guaranteed Boxed Localization (GBL) using anchors information is introduced. This method defines the localization problem as a constraint satisfaction problem set by mobility constraints and observations to anchors. The observation model is set using proximity constraints. However, the performance of GBL method is highly related to the number of anchors in the network.

In this paper, we propose a dynamic Interval-based Localization, using Anchors and Non-anchors information (ILAN). Each node is assumed to collect multi-hop information from all types of sensors. Nodes are thus able to communicate information from sensors, located outside their sensing range. With these properties, ILAN outperforms other techniques, especially in low-anchors density networks. The localization problem is resolved using interval analysis. Using this technique, the position estimation leads to rectangular areas, covering the real locations of the nodes. Compared to GBL, ILAN method yields more accurate estimates with low computational costs.

The rest of the paper is organized as follows. In Section II, we introduce the localization problem. In Section III, we first briefly present the theory of interval analysis. We then present the interval-based localization algorithm. The effectiveness of the method is illustrated in Section IV. Finally, Section V concludes the paper.

# II. PROBLEM STATEMENT

The localization problem consists of defining the nodes positions at every time step. The proposed method considers two types of sensors, anchors and non-anchor nodes. Anchors are equipped with Global Positioning Systems (GPS) [4] and thus they know their positions. The GPS-less sensors are called *non-anchor nodes* or simply *nodes*. They do not know their positions and thus need to be localized. Assume that the network is composed of  $N_a$  anchors and  $N_x$  nodes. Their coordinates

at time t are denoted respectively by  $a_i(t) = (a_{i,1}(t), a_{i,2}(t))$ and  $x_j(t) = (x_{j,1}(t), x_{j,2}(t))$ , where  $i \in \{1, ..., N_a\}$  and  $j \in \{1, ..., N_x\}$ . Besides measurements, the proposed method takes advantage of the mobility of the nodes, to estimate their positions. In the following, we first introduce the mobility model. We then present the observation model.

# A. Mobility model

Many mobility models have been proposed in literature to simulate the motions of the sensors [9]. In this paper, the localization method uses a general mobility model, where only the maximal velocity of the nodes,  $v_{max}$ , is assumed to be known. According to this model, the distance traveled by the nodes between two following time steps remains less than  $\Delta t.v_{max}$ , where  $\Delta t$  is the localization period. This leads to disk equations, centered on the previous positions of the nodes and having  $\Delta t.v_{max}$  as radius. Thus, the mobility model of a node  $x_j, j \in \{1, ..., N_x\}$ , is given by,

$$(x_{j,1}(t) - x_{j,1}(t-1))^2 + (x_{j,2}(t) - x_{j,2}(t-1))^2 \le \Delta t^2 \cdot v_{max}^2.$$
(1)

Any additional information about nodes motions could be used to refine the previous model.

## B. Observation model

At time t, each mobile node  $x_j$  receives signals from sets  $I_{a,j}(t) \subseteq \{1 \dots N_a\}$  of anchors and  $I_{x,j}(t) \subseteq \{1 \dots N_x\}$  of non-anchor nodes with specific Received Signal Strengths (RSSs), denoted respectively by  $\rho_{j,i}(t)$ ,  $i \in I_{a,j}(t)$ , and  $\rho_{j,k}(t)$ ,  $k \in I_{x,j}(t)$ . These RSSs are assumed to follow the Okumura-Hata model [10], given by,

$$\begin{cases} \rho_{j,i}(t) = \rho_0 - 10n_P \log_{10} \frac{d(\boldsymbol{x}_j(t), \boldsymbol{a}_i(t))}{d_0}, \\ \rho_{j,k}(t) = \rho_0 - 10n_P \log_{10} \frac{d(\boldsymbol{x}_j(t), \boldsymbol{x}_k(t))}{d_0}. \end{cases}$$
(2)

In (2),  $\rho_{j,i}(t)$  and  $\rho_{j,k}(t)$  are in dBm,  $\rho_0$  is the power measured (in dBm) at a reference distance  $d_0$ ,  $n_P$  is the path-loss exponent,  $d(\boldsymbol{x}_j(t), \boldsymbol{a}_i(t)) = \|\boldsymbol{x}_j(t) - \boldsymbol{a}_i(t)\|$  and  $d(\boldsymbol{x}_j(t), \boldsymbol{x}_k(t)) = \|\boldsymbol{x}_j(t) - \boldsymbol{x}_k(t)\|$  are the Euclidian distances between the considered node  $\boldsymbol{x}_j$  and the anchor  $\boldsymbol{a}_i$  or the node  $\boldsymbol{x}_k$  respectively.

In connectivity-based techniques, such as GBL ([6]), the RSSs are compared to a single threshold  $\rho_r$ , that is a function of the sensing range r of the node. To enhance the performance of the localization process in low-anchors density networks, we propose to use a hop counting technique using RSSs. For this reason, the RSSs are compared to several power thresholds  $\rho_r, \rho_{2.r}, \rho_{3.r}, ...$ , that are functions of the multiples of the sensing range r, 2.r, 3.r, ... The anchor i having  $\rho_{h_i.r} \leq \rho_{j,i} < \rho_{(h_i-1).r}$  is assumed to be a  $h_i$ -hop anchor. In other words,  $(h_i - 1).r < d(\mathbf{x}_j(t), \mathbf{a}_i(t)) \leq h_i.r$ . This leads to a ring, centered on the anchor  $\mathbf{a}_i$  and having  $(h_i - 1).r$  and  $h_i.r$  as internal and external radii. Similarly, the node k is a  $h_k$ -hop if  $\rho_{h_k.r} \leq \rho_{j,k} < \rho_{(h_k-1).r}$ . In such a case,  $(h_k - 1).r < d(\mathbf{x}_j(t), \mathbf{x}_k(t)) \leq h_k.r$ . Observation constraints

are thus given by,

$$\begin{cases} (h_i - 1)^2 \cdot r^2 < (x_{j,1}(t) - a_{i,1}(t))^2 + (x_{j,2}(t) - a_{i,2}(t))^2 \le h_i^2 \cdot r^2, \\ (h_k - 1)^2 \cdot r^2 < (x_{j,1}(t) - x_{k,1}(t))^2 + (x_{j,2}(t) - x_{k,2}(t))^2 \le h_k^2 \cdot r^2, \end{cases}$$
with  $i \in I_{a,j}(t)$  and  $k \in I_{x,j}(t).$ 

$$(3)$$

# III. INTERVAL-BASED LOCALIZATION

The localization problem is defined by both the mobility model (1) and the multi-hop observation model (3),

$$\begin{array}{l} (x_{j,1}(t) - x_{j,1}(t-1))^2 + (x_{j,2}(t) - x_{j,2}(t-1))^2 \leq \Delta t^2 \cdot v_{max}^2, \\ (h_i - 1)^2 \cdot r^2 < (x_{j,1}(t) - a_{i,1}(t))^2 + (x_{j,2}(t) - a_{i,2}(t))^2 \leq h_i^2 \cdot r^2, \\ (h_k - 1)^2 \cdot r^2 < (x_{j,1}(t) - x_{k,1}(t))^2 + (x_{j,2}(t) - x_{k,2}(t))^2 \leq h_k^2 \cdot r^2. \end{array}$$

with  $i \in I_{a,j}(t)$  and  $k \in I_{x,j}(t)$ . The resolution of this problem is performed using interval analysis. In the following, we first introduce the interval theory. We then develop the localization algorithm.

# A. Interval theory

The interval theory is a branch of mathematics that treats intervals as a new kind of numbers [7]. A real interval, denoted [x], is defined as a closed subset of IR as follows,

$$[x] = [\underline{x}, \overline{x}] = \{ x \in \mathbb{R} \setminus \underline{x} \le x \le \overline{x} \},$$
(5)

where  $\underline{x}$  and  $\overline{x}$  are the lower and higher endpoints of the interval. A box is a multidimensional interval defined by the cartesian product of real intervals,  $[\boldsymbol{x}] = [x_1] \times \cdots \times [x_n]$ .

An interval has a dual nature as both number and a set of real numbers. The interval theory takes advantage of this duality to extend set and arithmetic operations to intervals [7], [8]. Consider an operator  $\star \in \{+, -, *, \div\}$ . Then,

$$[x]\star[y] = [\min\{\underline{x}\star\underline{y}, \underline{x}\star\overline{y}, \overline{x}\star\underline{y}, \overline{x}\star\overline{y}\}, \max\{\underline{x}\star\underline{y}, \underline{x}\star\overline{y}, \overline{x}\star\underline{y}, \overline{x}\star\underline{y}\}].$$
(6)

Using these tools, we are able to define numerical problems in the interval framework. Solving these problems leads to guaranteed domains of solution.

#### B. Localization algorithm

The main idea of the proposed method is to consider the localization problem in the interval framework. The nodes positions  $x_j(t), j \in \{1, ..., N_x\}$ , are thus defined as two-dimensional boxes  $[x_j](t) = [x_{j,1}](t) \times [x_{j,2}](t)$ . Using interval coordinates, the localization problem is defined as a Constraint Satisfaction Problem (CSP). Resolving the problem consists of finding the minimal position boxes that satisfy all constraints. The mobility and observation equations, given in (1) and (3), are reformulated as follows,

$$([x_{j,1}](t) - [x_{j,1}](t-1))^2 + ([x_{j,2}](t) - [x_{j,2}](t-1))^2 = [0, \Delta t^2 . v_{max}^2],$$

$$(7)$$

$$([x_{j,1}](t) - a_{i,1}(t))^2 + ([x_{j,2}](t) - a_{i,2}(t))^2 = [(h_i - 1)^2 . r^2, h_i^2 . r^2],$$

$$([x_{j,1}](t) - [x_{k,1}](t))^2 + ([x_{j,2}](t) - [x_{k,2}](t))^2 = [(h_k - 1)^2 . r^2, h_k^2 . r^2],$$

$$(8)$$

with  $j \in \{1, ..., N_x\}$ ,  $i \in I_{a,j}(t)$  and  $k \in I_{x,j}(t)$ .

In order to solve the localization problem, different algorithms, called *contractors*, have been proposed [8]. Starting from an initial domain, the contractors consist of reducing its area in order to obtain the minimal box that encloses the



Fig. 1. Propagation phase.

solution. The contractor we are using in our method is the *forward-backward* algorithm. It consists of a simple algorithm that iterates all constraints, without any prior order, until no contraction is possible.

The solution of the problem consists of two phases at each time step, the propagation and the contraction phases. In the propagation phase, the nodes use their mobility equations (7) to propagate their previous boxes,  $[x_i](t-1)$ . Fig. 1 shows the first phase of the solution. The resulting boxes,  $[x_i]^*(t)$ , are then contracted using equations (7) and (8) in the forwardbackward contractor. During each iteration of the forwardbackward algorithm, each node exchanges location information with nodes and anchors within its communication range. The proposed approach is given in Algorithm 1. Note that since anchor positions are known, they are communicated to nodes only once at each time step, whereas non-anchor nodes exchange their coordinates at each iteration of the forwardbackward algorithm. Consider the localization problem of a mobile node  $x_1$  detecting a 1-hop anchor  $a_1$ , a 2-hop anchor  $a_2$  and a 2-hop mobile node  $x_2$ . Representing the anchors constraints would lead to a disk centered on  $a_1$  and having r as radius and a ring centered on  $a_2$  having r and 2r as radii respectively. The propagation of the position box  $[x_2]$ of  $x_2$  using the ring equation having r and 2r as radii leads graphically to an area having the shape of a distorted ring. This area covers all the points obtained by the propagation of any point of  $[x_2]$  using the ring equation. In other words, the node constraint leads graphically to the union of the rings centered on all the points of  $[x_2]$  and having r and 2r as radii. Fig. 2 shows this area. Using these constraints, in addition to the mobility constraint, in the forward-backward algorithm would lead to the minimal box covering the intersection area of all domains, as shown in Fig. 3.

# IV. SIMULATIONS

In order to evaluate the performance of our method, we first deploy the sensors in a  $100m \times 100m$  square area. We suppose that all sensors are able to communicate with each others. The sensing range r of the sensors is fixed to 10m. Sensors are assumed to move according to a group mobility model. In other words, all sensors are moving according to the same reference trajectory. Fig. 4 shows 10 nodes and 5 anchors moving around a sinusoidal trajectory according to the reference point group mobility model given in [9]. The maximal velocity  $v_{max}$  of the nodes is equal to 2.2719 $m.s^{-1}$ , whereas the localization period is set to 1s. In the following,



Fig. 2. Propagation of a box using a ring equation.



Fig. 3. Contraction phase.

we first evaluate the proposed method (ILAN). We then compare it to the interval-based technique GBL using connectivity measurements to anchors. We finally compare our method to a Monte Carlo-based approach. Note that all simulations are performed on Matlab 6.1, installed on an Intel Core2 Duo CPU.

### A. Evaluation of the ILAN method

In this section, we evaluate the effectiveness of the proposed method. We thus apply it to the network of Fig. 4. We assume at first that all sensors are able to communicate with each others, and that both nodes and anchors are considered in the

$$\begin{array}{l} \mbox{Input: } v_{max}, r, \mbox{observations} \\ \mbox{for } t \leq T \ \mbox{do} \\ \mbox{for } t \leq T \ \mbox{do} \\ \mbox{for } x \in T_{a,j}(t) \ \mbox{for } x = T \ \mbox{for } x_{j,2}](t) = [x_{j,2}](t) \cap ((a_{i,1}(t) - [b_{j,i,1}](t)) \cup (a_{i,1}(t) + [b_{j,i,2}](t))); \\ \mbox{for } x \in T_{x,j}(t) \ \mbox{do} \\ \mbox{for } x_{j} \leftarrow [x_{j,1}](t) = [x_{j,1}](t) \cap (([x_{k,1}](t) - [b_{j,k,2}](t))); \\ \mbox{for } x_{j} \leftarrow [x_{j,1}](t) = [x_{j,1}](t) \cap (([x_{k,1}](t) - [b_{j,k,1}](t))); \\ \mbox{[bj,k,2]}(t) = [x_{j,2}](t) \cap (([x_{k,2}](t) - [b_{j,k,2}](t))); \\ \mbox{for } x_{j} \leftarrow [x_{j,2}](t) = [x_{j,2}](t) \cap (([x_{k,2}](t) - [b_{j,k,2}](t))); \\ \mbox{for } x_{j} = (x_{j,2}](t) \cup ([x_{j,2}](t) - [x_{j,2}](t-1) - [b_{j,1}](t)) \cup ([x_{j,1}](t-1) - [b_{j,2}](t)) \cup ([x_{j,2}](t-1) + [b_{j,2}](t))); \\ \mbox{for } x_{j} = x$$

Algorithm 1: Interval-based localization using anchors and non-anchor nodes.

observation model. Fig. 5 shows the boxes obtained with our method for the 9-th mobile node. The average computational time per node per time step is equal to 0.027254s, whereas the average boxes areas is equal to  $675m^2$ . Let the estimation error be the distance between the center of the estimated box and the real position, then the average estimation error is equal to 11.8163m. Fig. 6 shows the average boxes areas in the top plot and the average estimation errors in the bottom plot for all the mobile nodes. The plot shows that the accuracy of the method varies from a node to the other. Indeed, the estimation accuracy depends on the distribution of other sensors around the considered node.

In order to illustrate the effectiveness of the use of the non-anchor nodes, we compare our method (ILAN) to an equivalent method using only anchors (ILA). We thus use deploy 5 anchors and 10 nodes in the network. All sensors are then moved using the same reference trajectory as in Fig. 4. Fig. 7 shows the boxes obtained with both methods for the 1-



Fig. 4. A group mobility of 10 nodes and 5 anchors moving around a common reference trajectory.



Fig. 5. Position boxes obtained with ILAN for the 9-th mobile node.

st node. The plot shows that estimation is more accurate with the ILAN method. Indeed, the average boxes area is equal to  $273.419m^2$  with ILAN whereas it is equal to  $340m^2$  with ILA; and the average estimation errors are equal to 8.6053m and 11.0279m respectively. This difference depends as well on the number and the disposition of the sensors in the network.

## B. Comparison to the GBL method

In this section, we compare our method (ILAN) to the Guaranteed Boxed Localization (GBL) technique proposed in [6]. Using intervals, the GBL method uses connectivity measurements of only anchors. In other words, each node in GBL uses only position information of anchors located within its sensing range to estimate its position. We first use a network having 5 anchors and 10 nodes. Fig. 8 shows the boxes obtained with ILAN and GBL method for the 9-th node. The plot shows that our method is much more accurate than GBL method in low-anchors density networks, at the cost of the computational time which increases from 0.000483*s* 



Fig. 6. Average boxes areas (in the top plot) and average estimation errors (in the bottom plot) for all the mobile nodes.



Fig. 7. Position boxes obtained with both ILAN and ILA methods for the 1-st mobile node.



Fig. 8. Positions boxes obtained with both ILAN and GBL methods for the 9-th mobile node with 5 anchors.



Fig. 9. Ratios of boxes areas (in the top plot) and ratios of estimation errors (in the bottom plot) ILAN/GBL for all the mobile nodes.

(GBL) to 0.0274s (ILAN). Fig. 9 shows the ratios of boxes areas in the top plot and the ratios of estimation errors in the bottom plot (ILAN over GBL).

Assume now that the network is composed of 30 anchors and 10 nodes. Fig. 10 shows the boxes obtained with the GBL and the ILAN methods. It is obvious that the accuracy of the GBL method increases with more anchors. However, it remains less accurate than our method, with less computation time (0.001232s with GBL, 0.13265s with ILAN). The average ratios of boxes areas and estimation errors (ILAN/GBL) are equal to 0.0994 and 0.3775 per node per time step, respectively.

## C. Comparison to the MCB method

The MCB method given in [5] is a Monte Carlo-based approach. At each time step, it generates a given number of positions, called *particles*, to cover the solution area. In order to compare our method to MCB, we use 30 anchors and the number of particles is set to 50. Fig. 11 shows the position boxes of one of the nodes obtained with our method. It shows the obtained particles with MCB, as well. The average computational times are equal to 0.13646s with our method and 1.5761s with MCB; whereas the average estimation errors are equal to 3.12m and 11.8304m respectively. Note that the boxes obtained with our method contain the real positions at each time step while the MCB particles do not cover correctly the solution areas at most of the steps. More particles are needed in order to improve the accuracy of MCB results, which leads to a higher consumption of energy, memory and time.

#### V. CONCLUSION

In this contribution, we presented an online guaranteed localization method for mobile sensor networks. The proposed technique uses multi-hop measurements involving both anchors and non-anchor nodes. The approach could thus be implemented in low-anchors density networks. Using interval analysis, this technique leads to location boxes where the real



Fig. 10. Positions boxes obtained with both ILAN and GBL methods for the 9-th mobile node with 30 anchors.



Fig. 11. Comparison of our method (ILAN) to the MCB method.

positions surely exist. The simulation results show that the proposed method overcomes the existing methods in terms of accuracy with low computational costs. In future works, we will deal with the localization problem under inaccurate environment and sensor failure assumptions.

### REFERENCES

- I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "Wireless sensor networks: A survey," *Computer Networks*, vol. 38, pp. 393–422, 2002.
- [2] "Special issue on self-organizing distributed collaborative sensor networks," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 4, 2005.
- [3] "Special issue on distributed signal processing in sensor networks," *IEEE Signal Processing Magazine*, vol. 16, 2006.
- [4] B. Hofmann-Wellenhof, H. Lichtenegger, and J. Collins, "Global positioning system: Theory and practice," *Springer-Verlag*, 1994.
- [5] A. Baggio and K. Langendoen, "Monte-carlo localization for mobile wireless sensor networks," *Second international conference on mobile ad hoc and sensor networks (MSN'06)*, 2006.
- [6] F. Mourad, H. Snoussi, F. Abdallah, and C. Richard, "Guaranteed boxed localization in manets by interval analysis and constraints propagation techniques," *IEEE Globecom*, 2008.

- [7] R. E. Moore, Methods and applications of interval analysis. Siam.
- [8] L. Jaulin, M. Kieffer, O. Didrit, and E. Walter, *Applied interval analysis*. Springer, 2001.
- [9] T. Camp, J. Boleng, and V. Davies, "A survey of mobility models for ad hoc network research," *Wireless Communications and Mobile Computing: Special issue on Mobile Ad Hoc Networking: Research, Trends and Applications*, vol. 2, pp. 483–502, 2002.
- [10] A. Medeisis, and A. Kajackas, "On the Use of the Universal Okumura-Hata Propagation Prediction Model in Rural Areas," *Proc. of Vehicular Technology Conference*, vol. 3, pp. 1815–1818, 2000.