Guaranteed Boxed Localization in MANETs by Interval Analysis and Constraints Propagation Techniques

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***Abstract*—In this contribution, we propose an original algorithm for self-localization in mobile ad-hoc networks. The proposed technique, based on interval analysis, is suited to the limited computational and memory resources of mobile nodes. The incertitude about the estimated position of each node is propagated in an interval form. The propagation is based on a state space model and formulated by a constraints satisfaction problem. Observations errors as well as anchor nodes imperfections are taken into account in a simple and computational-consistent way. A simple Waltz algorithm is then applied in order to contract the solution, yielding a guaranteed and robust online estimation of the mobile node position. Simulation results on mobile node group trajectories corroborate the efficiency of the proposed technique and show that it compares favorably to particle filtering methods.**

I. Introduction

Mobile Ad-hoc sensor NETworks (MANETs) is an emergent multidisciplinary research field. The attractiveness of such research field is essentially due to both its theoretical and applicative challenging aspects. MANETs are commonly defined as networks composed of low-cost, tiny and densely distributed mobile wireless sensor nodes, equipped with computational resources. The major constraints of such embedded devices are their limited memory, computational capabilities and energy reserve. In fact, the nodes batteries are not renewable and have thus a limited lifetime. Prolonging the lifetime of the whole network by designing collaborative and energy-aware processing tasks is the main challenging aspect in wireless sensor network research.

In mobile sensor networks, where the nodes mobility is uncontrolled, self-localization represents a fundamental issue. In fact, the success of data processing and decision-making is tightly related to the accuracy of the geographic locations of the deployed nodes. Target tracking or spatial interpolation for region monitoring [1] are illustrative examples of location-dependent applications of wireless sensor networks. Providing all nodes with Global Positioning Systems (GPS) [2] is a simple solution. However, given the high energy consumption, the high cost and the binding size of GPS systems, this technology is unsuitable in the wireless sensor network context. More reasonable localization methods may rely on providing few nodes (*anchors*) with GPS while designing efficient self-localization techniques for the remaining nodes.

The objective of this work is to deal with self-localization of mobile sensor nodes. Some existing works propose to perform successive static localization algorithms [3], [4]. The limited performances of such approaches are essentially due to the absence of any mobility modeling in the localization procedure. Recently, the Monte-Carlo Localization algorithm [5] and its enhanced version, the Monte-Carlo localization Boxed algorithm [6], have been proposed. This new kind of Bayesian filtering algorithms, based on state space models, aim at incorporating a mobility propagating model in the localization process. Because of the non linearity of both the mobility model and the observations, the Bayesian localization algorithm is implemented by an approximate sequential Monte Carlo (particle filter) method [7] where many samples (particles) are drawn in order to estimate the node position. However, in order to achieve good localization performances a high number of particles is needed.

In this paper, we propose a new localization technique based on a state space model and allowing the propagation of the position incertitude in an interval form. Tools of interval analysis [8] are then used in order to solve a constraint satisfaction problem where the prior mobility model together with the observations are considered as simultaneous constraints. Propagating boxes (multidimensional intervals) allows a guaranteed estimation of the node position using only few parameters (endpoints of one box). The proposed boxed localization achieves a substantial gain in both memory and localization computational cost while ensuring the same performances as the particle filter algorithms. Furthermore, a boxed modeling of the anchors positions and the sensing range incertitude enhances the robustness of the localization technique.

The paper is organized as follows. Section II is a brief introduction to interval analysis tools and their use in solving constraints satisfaction problems. Section III contains the main contribution of this paper: A guaranteed energy/memoryaware self-localization technique for MANETs. In Section IV, numerical results, illustrating the effectiveness of the proposed technique, are discussed. Section V concludes the paper.

II. Interval analysis

Interval analysis represents a rigorous mathematical tool aiming at manipulating intervals instead of real numbers. The interval framework provides an interesting alternative to punctual approximation, yielding guaranteed regions involving the correct solution. In addition, it allows to efficiently deal with problems involving interval data. In the following, we briefly recall the basics of interval analysis and constraints satisfaction tools.

A. Definitions and notations

A real interval, denoted $[x]$, is defined as a closed and connected subset of $\mathbb{R}$:

$$[x] = [\underline{x}, \overline{x}] = \{x \in \mathbb{R}, \underline{x} \le x \le \overline{x}\} \quad (1)$$

where $\underline{x}$ and $\overline{x}$ are the (finite or infinite) interval endpoints. A multidimensional interval (a box) of $\mathbb{R}^{n_x}$ can be defined as a cartesian product of n_x intervals: $[x] = [x_1] \times [x_2] ... \times [x_{n_x}]$.

Standard set operations are naturally defined on intervals, such as equality $(=)$, inclusion $(\subset)$, intersection $(\cap)$ and convex union defined as follows,

$$[x] \cup [y] = [\underline{x}, \overline{x}] \cup [\underline{y}, \overline{y}] = [\min\{\underline{x}, \underline{y}\}, \max\{\overline{x}, \overline{y}\}] \quad (2)$$

Arithmetic operations are also extended to intervals. For instance, we list the following basic operations,

- $[x] + [y] = [\underline{x} + \underline{y}, \overline{x} + \overline{y}]$
- $[x] - [y] = [\underline{x} - \overline{y}, \overline{x} - \underline{y}]$
- $[x] * [y] = [\min\{\underline{x} * \underline{y}, \underline{x} * \overline{y}, \overline{x} * \underline{y}, \overline{x} * \overline{y}\}, \max\{\underline{x} * \underline{y}, \underline{x} * \overline{y}, \overline{x} * \underline{y}, \overline{x} * \overline{y}\}]$

The arithmetic operations on intervals may take advantage of some algebraic properties such as *associativity*, *commutativity* and *subdistributivity* $([x]([y] + [z]) \subseteq [x][y] + [x][z])$.

The main shortcoming of interval analysis tools is their incapacity to enclose with a minimal size box any set of solutions in $\mathbb{R}^{n_x}$. In order to circumvent this drawback, constraints satisfaction algorithms (also called contractors) are used. The rationale behind these techniques is to use the redundancy of equations in order to reduce the size of the enclosing box.

B. Satisfaction of constraints : Waltz algorithm

A Constraints Satisfaction Problem (CSP) $\mathcal{H}$ is defined by m equations $f_j(x_1, ..., x_n) = 0, j = 1 \ldots m$ linking the components of an n-vector $\boldsymbol{x}$. The constraints can be put in a vector form $\boldsymbol{f}(\boldsymbol{x}) = 0$. Satisfying the constraints consists of finding a region $\boldsymbol{S} \subset \boldsymbol{B}$ defined as $\boldsymbol{S} = \{\boldsymbol{x} \in \boldsymbol{B} \mid \boldsymbol{f}(\boldsymbol{x}) = 0\}$ and where $\boldsymbol{B}$ is an initial prior box enclosing the solution. In general, $\boldsymbol{S}$ is not a box. Solving the CSP problem in an interval analysis framework consists of finding the minimal box $[\boldsymbol{x}^*] \subset \boldsymbol{B}$ such that $\boldsymbol{S} \subset [\boldsymbol{x}^*]$.

A contractor is an operator providing the solution $[\boldsymbol{x}^*]$ of the CSP problem by contracting the initial box $\boldsymbol{B}$. Different techniques can be used for contraction. In this paper, we use the Waltz algorithm [9] which consists of iteratively contracting each constraint, without any prior order, until the contractor becomes inefficient. The contractions are based on primitive constraints propagation. By primitive constraints, we refer to arithmetic operations and standard functions (cos, exp, etc.). For further details about the Waltz algorithm, refer to [10].

III. Guaranteed Boxed localization

The proposed Guaranteed Boxed Localization (GBL) technique is based on propagating a set of constraints defined by the prior mobility model of the moving nodes and the information messages communicated by the neighboring anchors (moving or static nodes equipped with positioning systems). In this paper, non-anchors information are not used. Therefore, only the individual prior moving model is used and the problem is reduced to separable self-localization problems. In the following, without loss of generality, we focus on the localization of only one mobile node. The same algorithm is implemented in parallel on the remaining nodes.

A. Model equations

Mobile nodes are moving in a *2*-dimensional deployment area, with a maximal velocity fixed to v_{max}. This means that over one time step, the node moves within the v_{max}-radius disk, centered at the previous position.

Let $\overrightarrow{x}(t) = (x_1(t); x_2(t))$ be the coordinates of the mobile node at time t. The dynamic state space model that we use in the GBL technique is defined by the following equations,

$$\begin{cases} x_1(t) = x_1(t-1) + v.\cos(\theta) \\ x_2(t) = x_2(t-1) + v.\sin(\theta) \end{cases} \quad (3)$$

where, for simplicity, the duration between two time steps is assumed equal to $1s$. v and θ are respectively the constant node velocity and direction between instants $t-1$ and t. Additional information about the prior distribution of the mobile node positions can be added to refine the general mobility model described above.

At each time step, the moving node is collecting local connectivity measurements. A connectivity measurement consists of one-bit information provided by the anchors within its communication range (one-hop anchors).

Let M be the number of anchors in the deployment area and m be the index of a single anchor such that $m \in \{1, ..., M\}$. The observation equation is defined as follows,

$$y^m(t) = \begin{cases} 1 & \text{if } \|\overrightarrow{a}^m, \overrightarrow{x}(t)\| \le r \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

where $\overrightarrow{a}^m$ represents the coordinates of the m^{th} anchor, r is the communication range of the node and $\|\cdot, \cdot\|$ is the Euclidean distance between the two points.

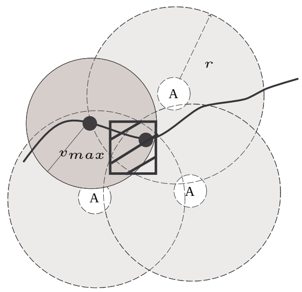

Fig. 1. Propagation and observation models with disk equations.

B. Localization by interval analysis

The interval framework provides an efficient and consistent methodology to solve the localization problem described by the equations 3 and 4 above. Instead of manipulating punctual positions, the pivotal idea of our approach is to associate the node position to a multidimensional interval (box). The position box covers a guaranteed rectangular area where all acceptable positions *certainly* exist.

The self-localization problem is formulated as a CSP problem, where a prior region **B** is contracted using Waltz algorithm and the constraints **f** set by the prior model and the observation equations.

Applying interval analysis to the above prior model disk, the equation 3 is formulated as follows,

$$([x_1](t) - [x_1](t-1))^2 + ([x_2](t) - [x_2](t-1))^2 = [0, v_{max}^2] \quad (5)$$

where $[x_1]$ and $[x_2]$ are the coordinate intervals of the mobile node and v_{max} is its maximal velocity.

In order to use interval analysis operators, the above constraint can be rewritten as follows,

$$\begin{cases} [x_1](t) = [[x_1](t-1) - [b_1](t)] \cup [[x_1](t-1) + [b_1](t)] \\ [x_2](t) = [[x_2](t-1) - [b_2](t)] \cup [[x_2](t-1) + [b_2](t)] \end{cases} \quad (6)$$

where $[b_1](t) = \sqrt{v_{max}^2 - ([x_2](t) - [x_2](t-1))^2}$ and $[b_2](t) = \sqrt{v_{max}^2 - ([x_1](t) - [x_1](t-1))^2}$.

The prior region is then contracted by the observed anchors connectivity constraints given by,

$$([x_1](t) - a_1^i)^2 + ([x_2](t) - a_2^i)^2 = [0, r^2], \; i \in I \quad (7)$$

where r is the communication range of the node, a_1^i and a_2^i are the coordinates of the i^{th} anchor and I is the set of all one-hop anchors detected by the mobile node.

Explicit equations are also needed to implement the Waltz algorithm. They are defined as follows,

$$\begin{cases} [x_1](t) = [a_1^i - [b_1^i](t)] \cup [a_1^i + [b_1^i](t)] \\ [x_2](t) = [a_2^i - [b_2^i](t)] \cup [a_2^i + [b_2^i](t)] \end{cases}, \; i \in I \quad (8)$$

where $[b_1^i](t) = \sqrt{r^2 - ([x_2](t) - a_2^i)^2}$ and $[b_2^i](t) = \sqrt{r^2 - ([x_1](t) - a_1^i)^2}$.

The correct position of the mobile node is situated inside the intersection of the mobility disk and the anchor disks. Applying the interval analysis consists of minimizing the box enclosing the disks intersection (see Fig.1).

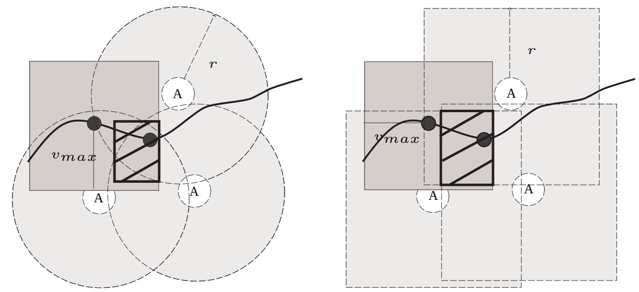

Fig. 2. Approximated schemes using square equations in the propagation model in (a) and (b) and in the observation model in (b).

In order to reduce the computational cost, we also propose two additional approximating schemes by relaxing the disk equations to square equations. In the first scheme, only the prior propagation equations are approximated as follows,

$$\begin{cases} [x_1](t) = [x_1](t-1) + [-v_{max}, v_{max}] \\ [x_2](t) = [x_2](t-1) + [-v_{max}, v_{max}] \end{cases} \quad (9)$$

Fig.2a shows the resulting box obtained by relaxing the propagation equations. In the second scheme, the connectivity constraints are also approximated as follows,

$$\begin{cases} [x_1](t) \subseteq [a_1^i - r, a_1^i + r] \\ [x_2](t) \subseteq [a_2^i - r, a_2^i + r] \end{cases}, \; i \in I \quad (10)$$

Fig.2b illustrates the box obtained using the approximations of the propagation and observation equations. Note that relaxing the constraints leads to larger positions intervals enclosing those obtained when using the exact disk constraints.

The interval-based approach provides also a convenient framework to deal with environment imperfections such as anchors positions or the communication range imprecisions. For instance, the GPS does not provide exact anchor positions. In the GBL technique, the uncertainty about these values can be incorporated by the use of intervals instead of approximated values. The constraints equations are modified as follows,

$$([x_1](t) - [a_1^i])^2 + ([x_2](t) - [a_2^i])^2 = [0, \max([r]^2)], \; i \in I \quad (11)$$

where $[a_1^i]$ and $[a_2^i]$ are the coordinates intervals of the i^{th} anchor and $[r]$ is the communication range box.

The explicit equations are then defined as follows,

$$\begin{cases} [x_1](t) = [[a_1^i] - [b_1^i](t)] \cup [[a_1^i] + [b_1^i](t)] \\ [x_2](t) = [[a_2^i] - [b_2^i](t)] \cup [[a_2^i] + [b_2^i](t)] \end{cases}, \; i \in I \quad (12)$$

where $[b_1^i](t) = \sqrt{[r]^2 - ([x_2](t) - [a_2^i])^2}$ and $[b_2^i](t) = \sqrt{[r]^2 - ([x_1](t) - [a_1^i])^2}$.

Remark 1: The proposed algorithm relies on contracting an initial box in order to cover the intersection of the propagated disk and the connectivity regions provided by the sensed anchors. The propagated disk is determined by the prior model maximal velocity whose value may affect the localization performances. In fact, an erroneous maximal velocity may lead to an inaccurate overlapping region: A small velocity (Fig.3a), with respect to the real value, may cause an empty intersection; while a larger value (Fig.3b) provides a vague information

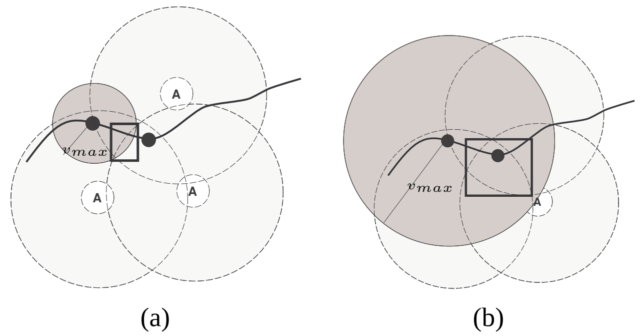

Fig. 3. Different values of v_{max} (lower in (a) and larger in (b) than its real value).

and thus, less contraction of the anchor overlapping region. In case of empty intersection, the mobility model constraint is relaxed and the Waltz algorithm is only applied on the observations constraints. A larger box (less accurate position) is thus obtained (see Fig.3b).

IV. SIMULATIONS

The performances of the proposed GBL localization technique in MANETs are tested using the reference point group mobility model given in [11]. According to this mobility model, the mobile nodes follow the same reference trajectory with small independent stochastic deviations. We consider 300 nodes deployed in a square $100m \times 100m$ region. The communication and sensing range are set to $10m$. The density of anchors is set in such a way that each node has at least 3 anchors in its vicinity. The reference trajectory is composed of 2 sinusoids with an abrupt change in order to test the capacity of the algorithm to track the nodes positions in difficult situations (see Fig.4). As the localization technique is only based on known anchor positions, the performances can be illustrated by following only one mobile node.

The average real velocity is around $v^* = 2.035m$ per second. The 3 proposed boxed localization techniques (with different prior and observations constraints types), are denoted as follows: case 1 (square-square), case 2 (square-disk) and case 3 (disk-disk). In order to compare the 3 cases, the maximal velocity parameter v_{max} is set to its average real value v^*. The computation times needed to accomplish the localization for the 3 cases are respectively $0.3590s$, $0.6390s$ and $0.7180s$ and the relative errors are respectively 1.98%, 1.78% and 1.78%. Fig.4 shows the estimated boxes obtained in the first and second cases respectively. The boxes obtained in case 2 are included in those of case 1 during the whole observation period.

In Fig.5, we show the ratios of the boxes areas obtained with the 3 cases. As can be expected, using the correct disk equations for observations contracts more the boxes. However, it is worth noting that relaxing the disk constraint to a square constraint for the prior mobility does not affect the obtained boxes (constant ratio equal to 1).

A. Variation of the maximal velocity

The estimated boxes result from the intersection of the prior and the observations model regions. Here, we show the

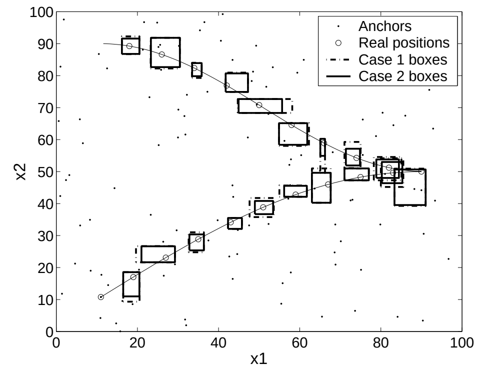

Fig. 4. Estimated boxes shown every $5s$ in case 1 (square-square) and case 2 (square-disk).

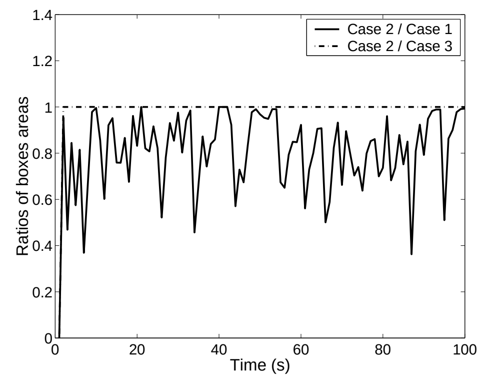

Fig. 5. Ratios of boxes areas for the considered 3 cases.

dependance of the localization outputs on the maximal velocity parameter v_{max}. In Fig.6, we report the average relative error as function of the ratio of v_{max} to its real value (from 0.5 to 6). The figure shows that the optimal results are obtained at the real maximal velocity value. After a certain value of v_{max}, the average error becomes constant. In fact, the prior region encloses the intersection of observation regions and has then no effect on contracting the boxes. This situation is equivalent to the absence of the prior model information. Fig.7 shows boxes obtained with v_{max} lower, equal and larger than its real value. With a lower parameter, boxes are reduced but do not contain the real positions at most of the time steps; while a larger parameter leads to larger boxes and thus a loss in estimation accuracy.

B. Comparison to Monte-Carlo boxed localization

The Monte-Carlo Boxed (MCB) localization algorithm [6] consists of two steps: (i) The prediction of particles inside the intersection of the mobility square and the observations approximated squares, and (ii) the filtering of particles by accepting only those respecting the disks constraints. These two steps are repeated until a fixed number of particles is kept. The estimated position is the mean of the particles. This method requires saving all the particles in the memory at every instant so that to be used in the next time step.

In order to compare the proposed Guaranteed Boxed Localization (GBL) to the MCB method, we use the same simulation conditions as above. The number of particles is set to 50 particles. In Fig.8, we plot the estimated boxes obtained by

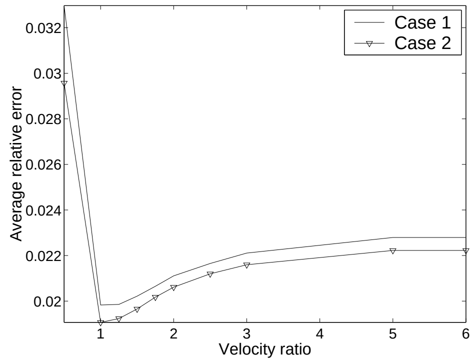

Fig. 6. Average relative error vs. velocity ratio.

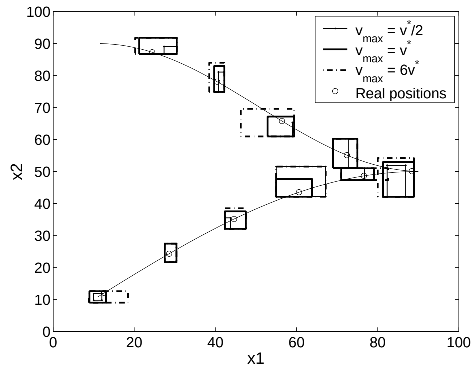

Fig. 7. Estimated boxes for three different values of v_{max}.

the GBL technique. We plot also the 50 particles obtained by the MCB method. Fig.8 clearly illustrates that with 50 particles, the real position can not be efficiently covered. The average relative error is equal to 2.25% with the MCB method while it is around 1.78% with the GBL method (with case 2). The time needed to accomplish the localization algorithms is around $0.6550s$ for the GBL method and $2.7720s$ for the MCB method. Besides the gain in computation time, MCB requires the storage of at least 50 particles every time step while GBL only needs to save the resulting endpoints coordinates describing one estimated box.

C. Boxed anchors positions and communication range

All anchor-based localization methods assume exact anchor positions and communication range information. Incertitude about these values may lead to erroneous localization results. In GBL method, using boxes instead of approximated values for these parameters, enhances the robustness of the localization technique. In order to illustrate this robustness, we compare GBL to MCB method using uncertain anchors positions and communication range. We put a $2m \times 2m$ box around the true anchors positions and we vary the communication range between $9m$ and $11m$ while $10m$ is the exact value. The average relative errors obtained are equal to 1.97% and 3.02% for GBL and MCB respectively, showing the capacity of the GBL technique to efficiently deal with uncertainty about the input parameters.

V. Conclusion

In this paper, we proposed a Guaranteed Boxed Localization (GBL) algorithm based on interval analysis for self-

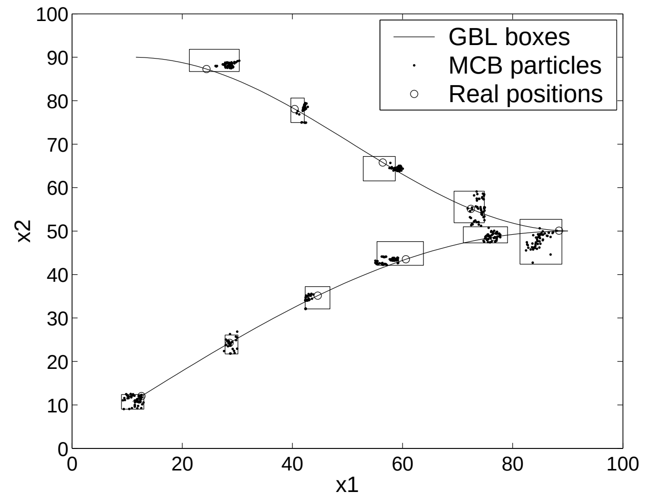

Fig. 8. Estimated boxes with GBL and estimated particles with MCB using 50 particles.

localization in MANETs. The dynamic localization, based on a state space model, allows a more accurate localization than running a repeated static localization. Associating boxes to estimated positions, one is able to cover bounded areas where solutions surely exist (guaranteed localization). Compared to Monte Carlo-based algorithms, the computation time and the needed memory are highly reduced while the total error is decreased. Future works will focus on the use of belief functions in order to deal more efficiently with inaccurate environments where erroneous information might be communicated between nodes.

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