ANCHOR-BASED DISTRIBUTED LOCALIZATION IN WIRELESS SENSOR NETWORKS

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ABSTRACT

In this paper, we introduce a distributed strategy for localization in a connected wireless sensor network composed of limited range sensors. Our distributed algorithm is computed through the network and provides sensor position estimation from local connectivity measurements. This work takes advantage of a conditionally and locally convex criterion that is easier to compute than the non-convex Kruskal's *Stress*. In addition, no initialization is required such as estimating distances between sensors and absolute reference positions. Our iterative technique is distributed among sensors and guarantees the minimization of a global cost function. It might be used in a Mobile Ad-hoc Network framework (*MANet*) thanks to its fast convergence, its low computational cost and its much higher accuracy compared to state-of-the-art methods.

Index Terms: localization, distributed signal processing, sensor network

1. INTRODUCTION

Recent technological advances in electronics and wireless communications have led to the development of tiny, low-power and active sensors for phenomenon observations purpose. Deploying randomly and densely a large number of low-cost devices in the environment of interest, designing an energy-appropriate routing scheme and implementing an efficient distributed algorithm through the sensor network seem to be opened to large opportunities, specially in monitoring and tracking applications [1]. The first step of estimating the sensor-location after deployment is thus a crucial issue. GPS system, which is a free service, may solve in practice our localization problem for each node of the embedded network. However, GPS receivers at each device may be too expensive and too power-intensive for the desired application, whose low energy consumption is the main constraint to respect [2]. As a consequence, we just consider few sensors, called *anchor nodes*, which have a perfect a priori knowledge of their coordinates thanks to GPS receivers.

Our work respects all these features which characterize wireless sensor localization problem: distributed processing, limited power, limited memory, limited energy reserve and limited computional capacities.

The RSSI technique -*Received Signal Strength Indicator*- can be used and allows us to estimate dissimilarity measurements between sensor nodes by log-degradation of the signal strength with distances and setting a received power threshold. Despite its relative poor accuracy, this technology is widespread because of its simple execution and its cheap cost [3].

This paper is organized as follows: In section 2, we briefly describe the problem statement. Section 3 presents the developed algorithm. In section 4, a probalistic interpretation of the localization criterion is given. In section 5, experimental results confirming the algorithm efficiency are shown.

2. PROBLEM STATEMENT

Consider a network of $N = m + n$ sensors, living in a p-dimensional space ($p = 2$ or 3 upon localization takes place in plane or space, with $n \gg m > p$). Let $\mathbf{x}_i \in R^p$ be the sensor i's coordinates, $\{\mathbf{x}_i\}_{i=1}^m$ is the set of anchor nodes coordinates which positions are known, $\{\mathbf{x}_i\}_{i=m+1}^N$ are the unknown remaining sensors coordinates. If we assume that maximum spotting sensor range is equal to the distance r, therefore sensor i will consider sensor j as a neighbour if and only if the distance $\|\mathbf{x}_i - \mathbf{x}_j\|$ is lower than r. However, two cases must be considered:

First, if sensors proximity measurements are connectivities (i.e, $\delta_{ij}^{con} = 1$ if the sensor i detects j, $\delta_{ij}^{con} = 0$ otherwise), Δ_{con} is similar to an adjacence matrix pertubated by detection inaccuracy. Therefore, the purpose is to estimate unknown coordinates $\{\mathbf{x}_i\}_{i=m+1}^N$ in a simple and distributed way, from local dissimilarities measurements δ_{ij}^{con}, namely connectivities, and anchors nodes coordinates $\{\mathbf{x}_i\}_{i=1}^m$.

In the second case, sensors proximity measurements are not connectivities but estimated distances by RSSI technique for instance within a certain range. The dissimilarity matrix $\Delta_{dis} = [\delta_{ij}^{dis}]_{i,j=1}^N \simeq [\|\mathbf{x}_i - \mathbf{x}_j\|_2]_{i,j=1}^N$ may be made by the Euclidean distances approximated between neighbour sensors (i, j). Thus, the goal is still the same: estimating unknown coordinates $\{\mathbf{x}_i\}_{i=m+1}^N$ thanks to a distributed process, from local distances measurements δ_{ij}^{dis} and anchors nodes coordinates $\{\mathbf{x}_i\}_{i=1}^m$.

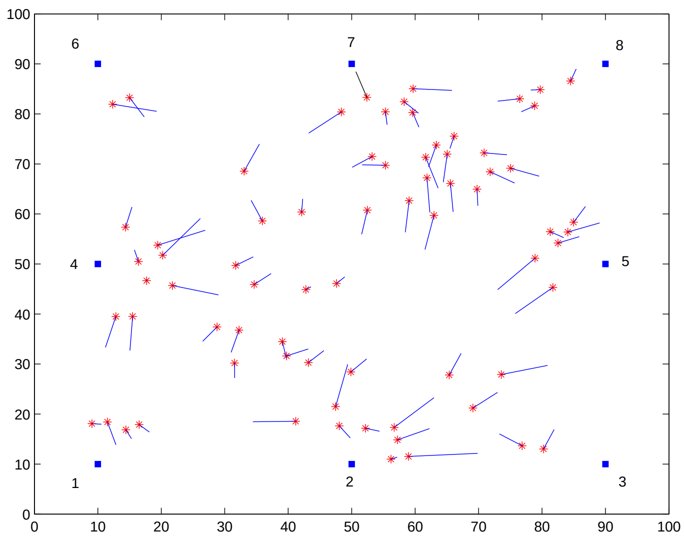

Fig. 1. Estimated positions of 72 sensors randomly spread over a 90 $\times$ 90 surface within a static framework. The network has 8 peripheral anchors, represented by squares. Only connectivity measurements were available. Device range was fixed to 20. Estimated locations of nodes are indicated by crosses. Distance errors with respect to true positions are symbolized by segments.

3. DEVELOPED ALGORITHM

The proposed algorithm, called *dG-Loc* (distributed Gamma-Localization), uses connectivity measurements and Gamma *a priori* distribution for computing sensor positions. Motivated by seeking a robust convex criterion, we compute sensor positions by minimizing the following cost function:

$$\mathbf{S}(X) = \sum_{i=m+1}^{N} \Big[\sum_{j\in\mathcal{V}(i)} \left(\|\mathbf{x}_i - \mathbf{x}_j\|^2 - \alpha_{r,\sigma} \log \|\mathbf{x}_i - \mathbf{x}_j\|^2 \right) \Big] \quad (1)$$

where $X = [\mathbf{x}]_{i=m+1}^{N}$ is the matrix of sensor positions to estimate, $\mathcal{V}(i)$ the set of neighbors of the i-th sensor, and $\alpha_{r,\sigma}$ a parameter that is dependent on sensor range r and detection accuracy σ. We suggest to use $\alpha_{r,\sigma} = r/2 - 2\sigma^2/r$ that will be argued in the next section.

A regularization term associated to anchor position helps the convergence and points the network (*see next section*):

$$\hat{\mathbf{x}}_i = \arg\min_{\mathbf{x}_i} \sum_{j\in\mathcal{V}(i)} \left(\|\mathbf{x}_i - \mathbf{x}_j\|^2 - \alpha_{r,\sigma} \log \|\mathbf{x}_i - \mathbf{x}_j\|^2 \right) + \lambda \|\mathbf{x}_i - \mathbf{x}_k\|^2 \quad (2)$$

where $\mathbf{x}_k$ is absolute coordinate of anchor node neighbor of node i and the weight λ sized beforehand.

Since $\mathbf{S}$ is a sum of local cost functions $\mathbf{S} = \sum_{i=m+1}^{N} \mathbf{S}_i$, a gradient descent performs minimization in a distributed manner. Minimization of $\mathbf{S}(X)$ is performed from sensor to sensor with $i = \{m+1...N\}$:

$$\hat{\mathbf{x}}_i = \arg\min_{\mathbf{x}_i} \mathbf{S}_i + \lambda \|\mathbf{x}_i - \mathbf{x}_k\|^2 \quad (3)$$

Cycles are repeated until convergence of the algorithm (see Fig. 1). The pseudo-code used for simulations in a static framework is given in Tab. 1 while the other one for the mobile sensor network case is given in Tab. 2.

Inputs: $\{\mathbf{x}_i\}_{i=1}^{m}, \mathcal{V}(i), m$
Initialization: compute randomly $\mathbf{x}_i$ for $i = \{m+1 \dots N\}$
for $c = 1$ **to** C
  for $i = m+1$ **to** N
  - find $\hat{\mathbf{x}}_i^{(c)}$ from equation (4) at node i
  - communicate $\hat{\mathbf{x}}_i^{(c)}$ to neighbors of node i
  end for
end for

Table 1. Pseudo-code for localization estimation in a static framework. The number of cycles C is equal to 5 on average which is sufficient for efficient accuracy and helps for high computing speed.

Inputs: $\{\mathbf{x}_i\}_{i=1}^{N}, \mathcal{V}(i), m$
Initialization: $t = 0$, apply *dG-Loc* as in a static framework
for $t = 1$ **to** T
  for i = m+1 **to** N
  - compute $\mathbf{x}_i$ according to a random motion
  - compute new connectivities of node i
  end for
  for $c = 1$ **to** C
    for $i = m+1$ **to** N
    - find $\hat{\mathbf{x}}_i^{(c)}$ from equation (4) at node number i
    - communicate $\hat{\mathbf{x}}_i^{(c)}$ to neighbors of node i
    end for
  end for
end for

Table 2. Pseudo-code for *MANet* framework of T motions. The number of cycles C after each motion is very low (2 or 3 cycles are generally sufficient).

4. PROBABILISTIC INTERPRETATION

Under a Bayesian framework, minimizing (1) can be considered as maximizing the *a posteriori* distribution associated with sensor location given the measurements of proximity:

$$\hat{X} = \arg\max_{X} p(X \mid E)$$

where E is the graph modelling the neighborhood relations. The *a posteriori* distribution $p(X \mid E)$ is given by Gamma density with mean $\frac{r}{2}$, which represents half of sensor range, and variance σ^2 depending on sensor capability. We justify this Gamma choice by its definition domain R^{+*} and simplicity of computing (see Fig. 2). The strict positiveness of the Gamma argument avoids the aggregation of the points to a single point.

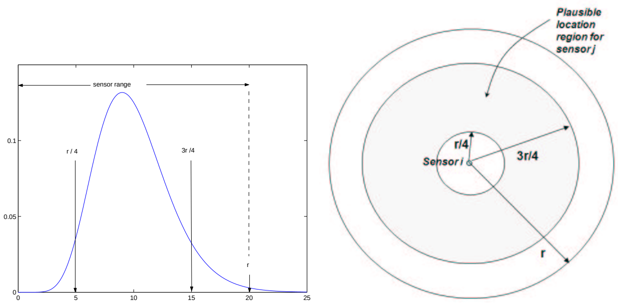

Fig. 2. The strong plausible region of location for node j (neighboor of node i) is shown shaded as regards to Gamma distribution shape. The parameter of the distribution correspond to $\mathbb{E}(\mathbf{x}) = \frac{r}{2} = 10$ and $\mathrm{V}(\mathbf{x}) = 10$ such as experimental conditions used for simulations.

We assume that neighbors of i-th node are independently identically Gamma distributed which can be parameterized in terms of a shape parameter k and rate parameter θ. The mean $\mathbb{E}(\mathbf{x})$ is fixed at half of sensor range i.e. $k\theta = \frac{r}{2}$ and the variance $\mathrm{V}(\mathbf{x}) = k\theta^2$ equal to σ^2. The sensor positions are thus derived as solution of the following optimization process:

$$\begin{aligned}
\hat{\mathbf{x}}_i &= \arg\max_{\mathbf{x}_i} \log f(\{\mathbf{x}_i\}/\{\mathbf{x}_j\}_{j\in\mathcal{V}(i)}) \\
&= \arg\max_{\mathbf{x}_i} \log \prod_{j\in\mathcal{V}(i)} \Gamma(\|\mathbf{x}_i - \mathbf{x}_j\|^2; k; \theta) \\
&= \arg\max_{\mathbf{x}_i} \sum_{j\in\mathcal{V}(i)} \left(\theta(k-1)\log\|\mathbf{x}_i - \mathbf{x}_j\|^2 - \|\mathbf{x}_i - \mathbf{x}_j\|^2\right) \\
&= \arg\min_{\mathbf{x}_i} \sum_{j\in\mathcal{V}(i)} \left(\|\mathbf{x}_i - \mathbf{x}_j\|^2 - \alpha_{r,\sigma}\log\|\mathbf{x}_i - \mathbf{x}_j\|^2\right) \\
&= \arg\min_{\mathbf{x}_i} \mathbf{S}_i
\end{aligned}$$

where $\alpha_{r,\sigma} = r/2 - 2\sigma^2/r$.

It is clear that the global criterion is not necessarily convex. The regularization term associated to anchor position is thus used to incorporate more informations in the process and avoid inherent transformations (reflexion, rotation...). This additional term can be considered as *a priori* information on sensors situated close to an anchor:

$$\hat{\mathbf{x}}_i = \arg\min_{\mathbf{x}_i} \mathbf{S}_i + \lambda\|\mathbf{x}_i - \mathbf{x}_k\|^2$$

where the weight λ can be viewed as the inverse variance of the Gaussian *a priori* distribution with mean $\mathbf{x}_k$.

5. EXPERIMENTS

Since our approach does not require estimating node-anchor distances, we are not concerned with *flooding information problem* which is very energy-expensive for large networks, even for low anchor density. Therefore, we avoid being tricked by this scaling problem, contrary to state-of-the-art distributed localization algorithms such as *Ad-hoc positioning* [4], *Robust positioning* [5] and *N-hop multilateration* [6].

Before comparing with other models, we try to evaluate the performance of our algorithm in a special case where best and worst mean error can be evaluated geometrically. Considering the configuration such as the one in Fig 1, 8 peripheral anchors and 72 sensors which location has to be estimated are deployed in a 90×90 area. The detection range is fixed to 20. Our ideal case considers that anchor sensors are always situated in the periphery and non-anchor sensor are regullary spread (i.e. neighboors are equidistant from each other) in order to uniformly cover all the surface. Thereby, each sensor is situated at the center of a 10×10 square surface. To evaluate geometrically the highest and lowest error at any node, this configuration supposes that all neighboors are fixed to their true location. Potential points situated inside the gray area (see Fig. 3) do not change the gradient of any exclusively connectivity based criterion (without a priori information). Thus, the best error that can be observed is 0 and the worst error is 7.32 in this particular case. We test our *dG-Loc* algorithm on this special configuration, with initial random configurations. We obtain a mean distance error per sensor, over 300 simulations, equal to 0.52. This result ($0.52 \ll 7.32$) confirms that the proposed algorithm achieves good results considering that the dead zone $[0; 7.32]$ is calculated in advantageous hypothesis (knowing the exact location of neighboor).

We compare our *dG-Loc* algorithm with the distributed method *dwMDS* [7], which needs peripheral anchors and RSSI technique too. The *dwMDS* algorithm locally minimizes the *Stress* criterion defined as $R(\mathbf{x}_i) = \sum_{j\in\mathcal{V}(i)}(\delta_{ij}^2 - \|\mathbf{x}_i - \mathbf{x}_j\|^2)^2$. For comparative experimentations, node locations were randomly generated over a square surface 90×90, each configuration being used by the two methods. Sensor range r was fixed to 20, so that many sensors were anchor-blind. Detection accuracy σ was fixed to $\sqrt{10}$. The *dwMDS* algorithm performs a gradient descent at each iteration on a four-degree polynomial, which significantly increases its computation complexity compared to our method. In addition, *dwMDS* can be entrapped into local minima. Three hundreds simulations with 8 cycles each were run. CPU allocation mean time for *dwMDS* was equal to 130.17 seconds per cycle, and 39.79 seconds per cycle for our algorithm (Matlab 6.1, CPU 3.40 GHz, 1.00 Go RAM). Fig. 4 compares convergence speed and steady-state error of both algorithms. The mean distance error per sensor, averaged over 300 configurations, is presented in Tab. 3. Simulations were also per-

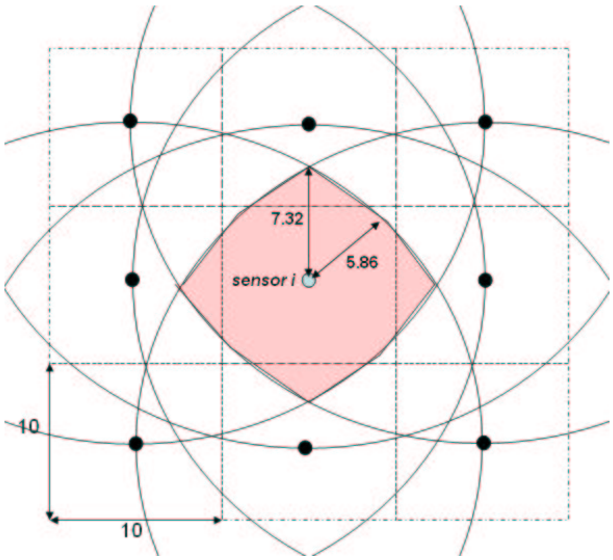

Fig. 3. The area colored in gray represents the feasible region for the sensor i where connetivities constraints with its 8 neighboors are respected (range detection is fixed to 20).

formed with moving sensors in order to test the accuracy and the tracking capabilities of our algorithm (see Fig. 5).

	Error mean	Standard deviation
dwMDS	**7.47**	**1.32**
dG-Loc	**5.11**	**1.82**

Table 3. *dwMDS* and *dG-Loc* performances comparaison.

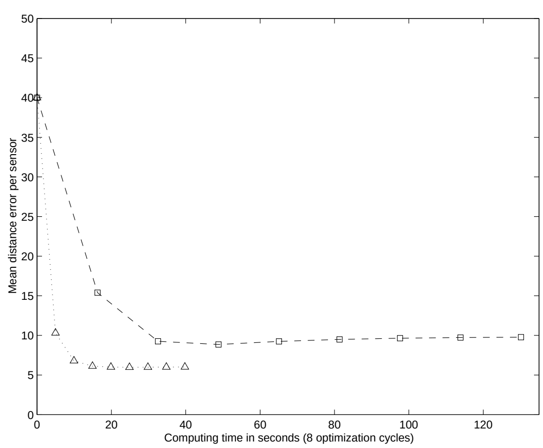

Fig. 4. Convergence of mean distance error per sensor as a function of computation time. (*dG-Loc*: triangles; *dwMDS*: squares). Each triangle/square corresponds to a cycle.

In order to test the robustness of *dG-Loc* algorithm, the connectivities measurements between non-anchor nodes are noised. In practice, a threshold on the received power associated to the sensor range is fixed. If the signal power of the

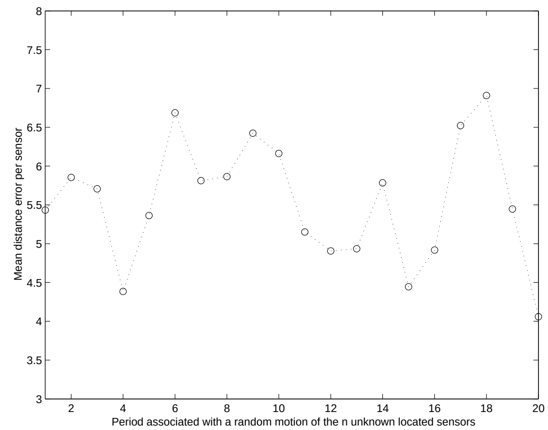

Fig. 5. Evolution of mean distance error per sensor, within a mobile framework. Random moving vectors for each sensor are distributed according to a Gaussian function with mean 8 and variance equal to 1, and motions are strictly allowed inside the experimental surface.

sensor j received at sensor i is higher than the threshold, the neighboor relation between node i and j is admitted. Otherwise, the maximum spotting sensor range r is considered surpassed and sensor i and j are not connected. However, it has to be considered that detection measurement accuracy degrades with distance on RSS receiver [7]. Thus, we consider that a sensor i detects a neighboor node j with a probability 1 if $\|\mathbf{x}_i - \mathbf{x}_j\| < \frac{r}{2}$. Nevertheless, there is a probability of non-detection equal to 0.2 when $\frac{r}{2} \leq \|\mathbf{x}_i - \mathbf{x}_j\| \leq r$ i.e. δ_{ij}^{con} takes the value 0. The probability of false alarm which consists in detecting a non-neighboor sensor, distanced by $\frac{3r}{2}$ to the maximum, is fixed to 0.1 (δ_{ij}^{con} takes the value 1 despite $r < \|\mathbf{x}_i - \mathbf{x}_j\| \leq \frac{3r}{2}$). The mean distance error per sensor with noised connectivities measurements at each cycle, averaged over 300 simulations, is presented in the Fig. 6. Performance results are shown for different N values (number of nodes). We note that localization performances are significantly degraded for low densities.

6. CONCLUSION

In this paper, we have shown that our algorithm offers interesting performance compared to a state-of-the-art models. This model offers good opportunities with incorporing inter-distances measurements instead of connectivities in the mean Gamma distribution. We are more interested now in investigating kernel approaches which offers a non-linear denoising process and is strongly related to *Metric MDS* resolution. Preliminary results show that performances are better when proximities measurements are highly corrupted such as connectivities.

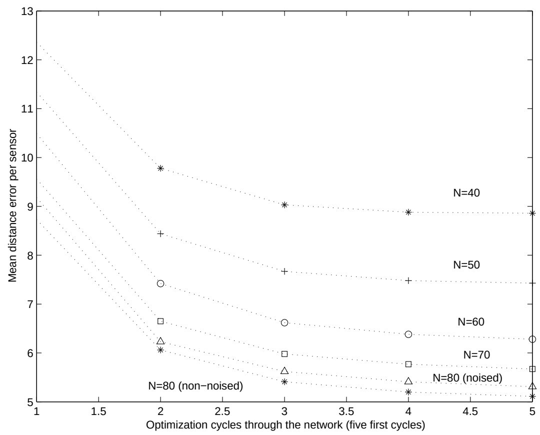

Fig. 6. Evolution of mean distance error per sensor function of network density ($m = 8$, $s = 20$, N varying)

7. REFERENCES

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